On the Use of Time Frequency Analysis for Adaptive Target Detection in Highly Textured Monodimensional SAR Images

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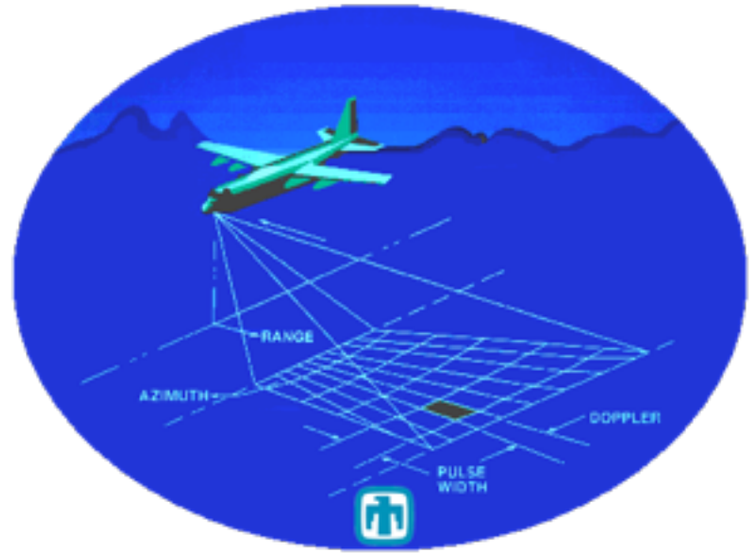
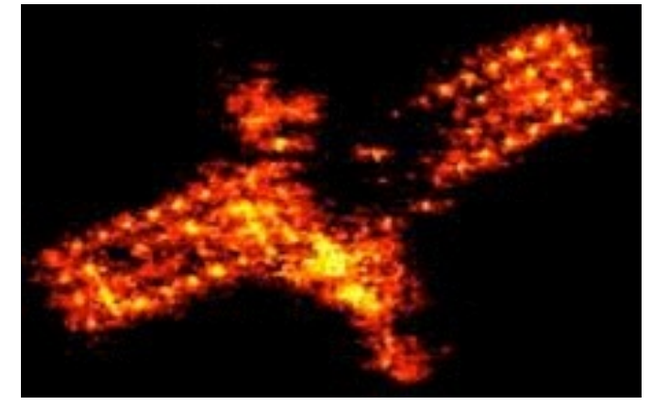
Outline

- **SAR Imaging**
  - Conventionnal model - Applications
  - Limitation of this model
  - Dispersivity and anisotropy of the SAR scatterers
- **Detection**
  - in mono-channel SAR image
  - in multi-channel SAR image
  - Problems related to SAR image
- **Time-Frequency for SAR**
- **Advanced Robust Detection and Estimation**
- **Application to Detection in SAR Image**
- **Conclusion**

Radar/SAR Imaging

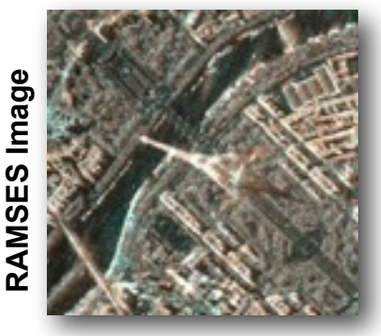
ONERA RAMSES Image

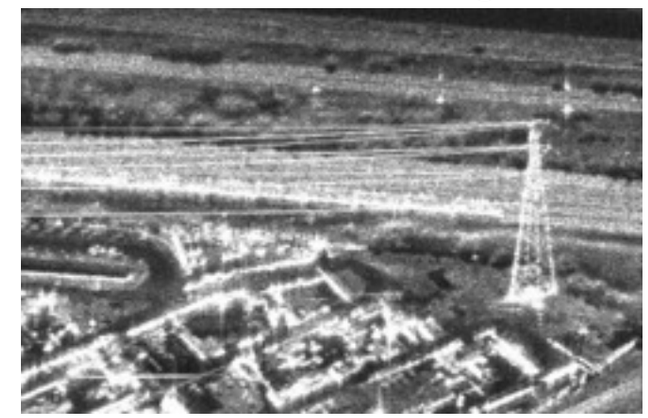




ONERA ISAR Image

RAMSES Image





ONERA RAMSES Image

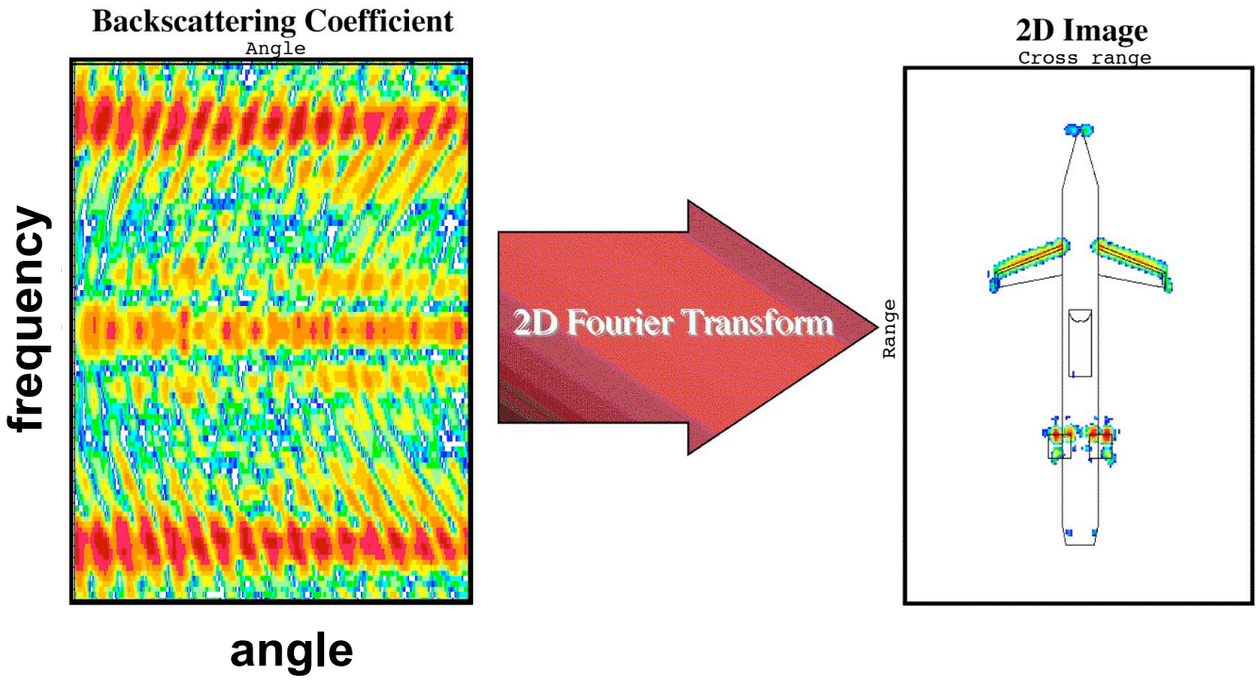
Radar Imaging allows to build more and more precise images :

- **Current use of very high bandwidth and long integration time (high azimuth bandwidth): Very high spatial resolution (< 10cm),**
- **Application to surveillance (detection, change detection), classification, 3D reconstruction, EM analysis, …**
- **Due to the growing complexity of the scene (non stationarity, non-Gaussianity), need to derive new procedures to exploit these images.**

Conventional Principle of Radar/SAR Imaging

Conventional Fourier Imaging (laboratory, SAR, ISAR) :

- **Assumptions of white and isotropic bright points,**
- **does not exploit the potential non-stationarity of the scatterers.**



- Hypothesis of bright points model: all the reflecting elements of the scene localized in $\mathbf{x}$ and characterized by the spatial repartition function $A(\mathbf{x})$ have **the same behavior** for any wave vectors $\mathbf{k} = \dfrac{2\,f}{c}\,(\cos\theta, \sin\theta)^T$. The backscattering coefficient $H(\mathbf{k})$ acquired by the radar takes the form:

$$H(\mathbf{k}) = \int_{\mathcal{D}_{\mathbf{x}}} A(\mathbf{x})\, e^{-2\,i\,\pi\,\mathbf{k}^T\,\mathbf{x}}\, d\mathbf{x}$$

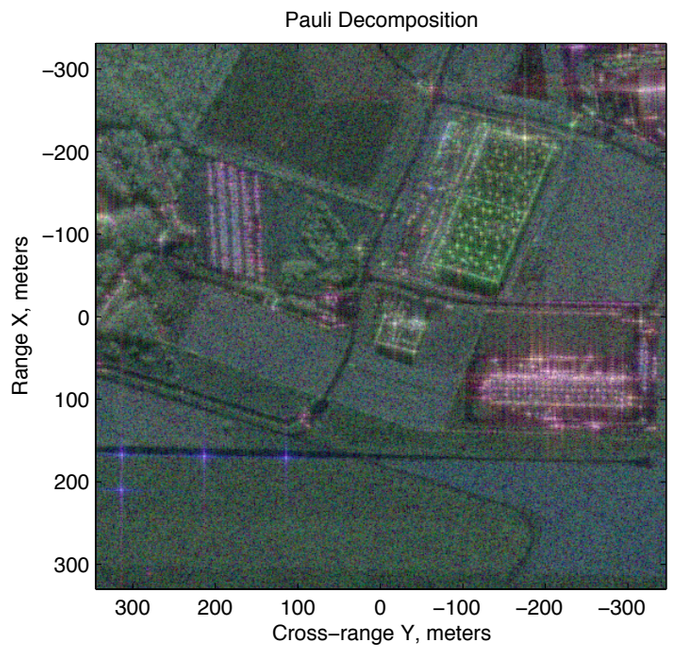
- The construction of the radar spatial image $A(\mathbf{x})$ is then given by the inverse classical Fourier transform of the backscattering coefficient $H(\mathbf{k})$:

$$A(\mathbf{x}) = \int_{\mathcal{D}_{\mathbf{k}}} H(\mathbf{k})\, e^{2\,i\,\pi\,\mathbf{k}^T\,\mathbf{x}}\, d\mathbf{k}$$

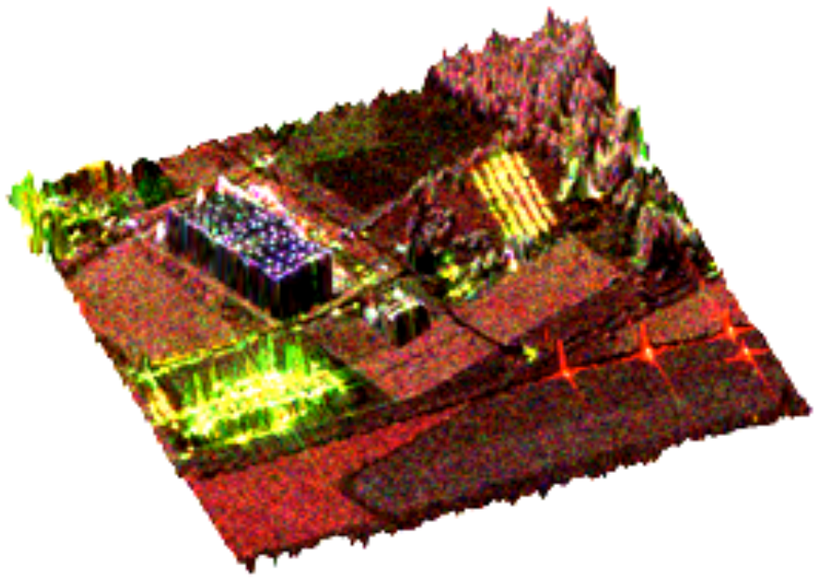
Examples of Applications in Multi-Channel SAR Image

For multichannel SAR Images, each pixel of the spatial image is associated to a vector of important and useful informations:

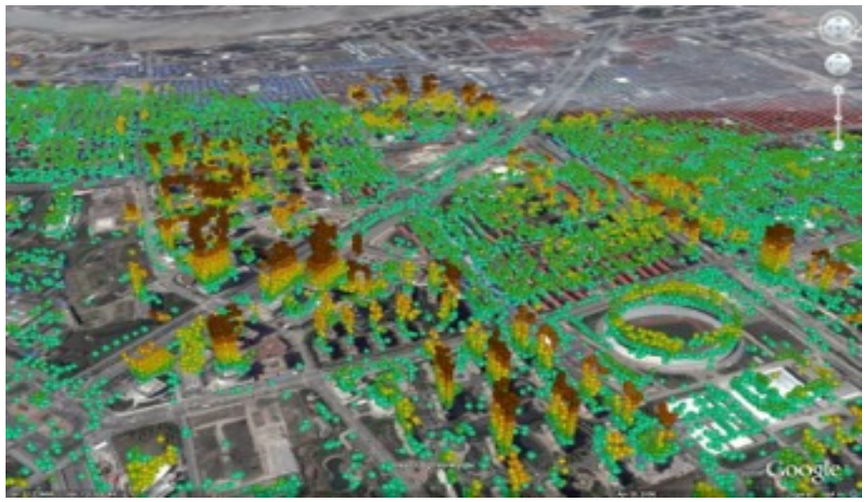
- **polarimetric channels (POLSAR),**
- **interferometric channels (INSAR),**
- **polarimetric and interferometric channels (POLINSAR),**
- **Multi-temporal, multi-passes SAR Image,**



EM behavior of the terrain in POLSAR images



Estimation of the height in POLINSAR images



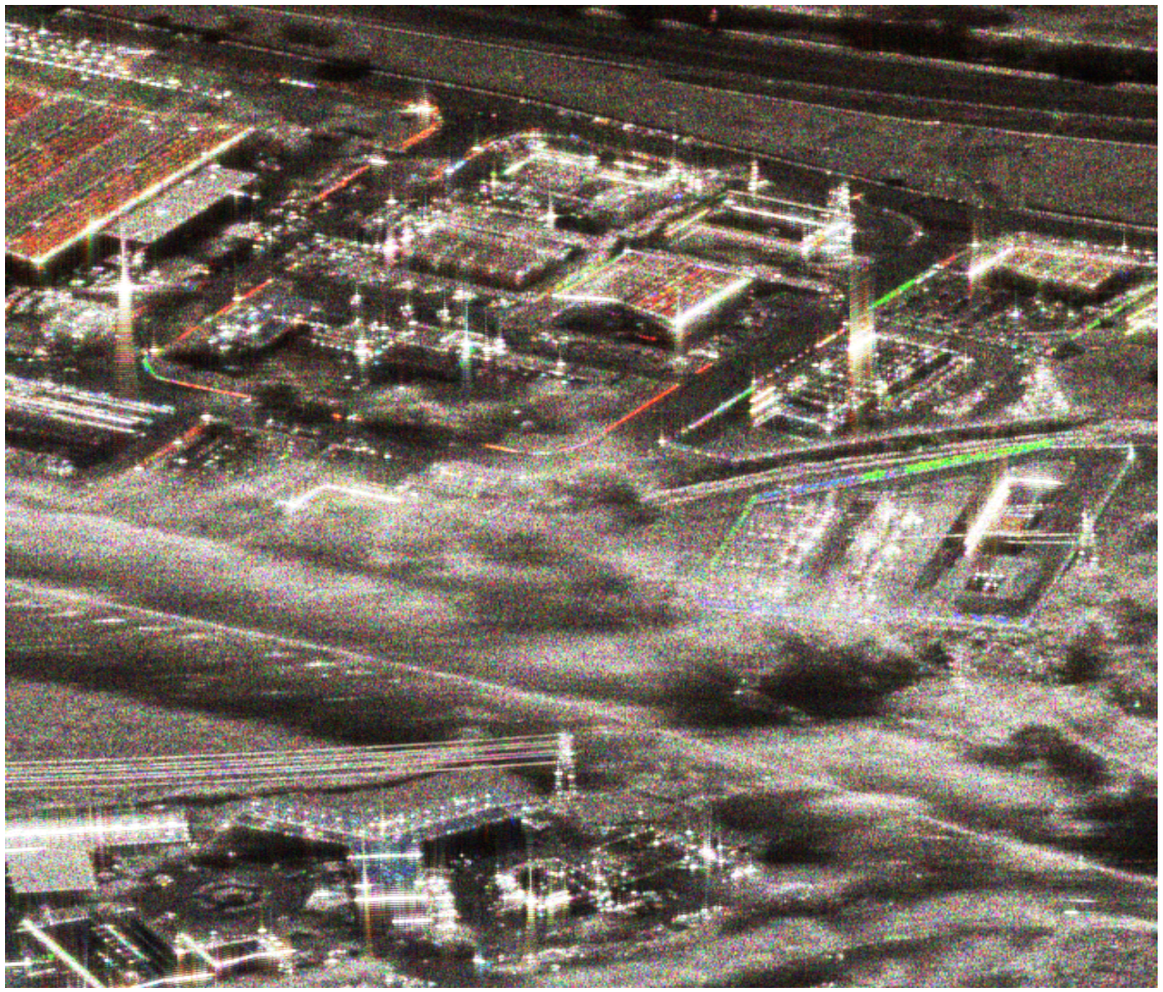
Analysis of the structures displacement in Shangai with multi-temporal SAR images (@Telespazio)

Almost all the conventional techniques of detection, parameters estimation and classification in multichannel SAR images are based on the multivariate Gaussian statistic with additional hypotheses of stationarity and homogeneity.

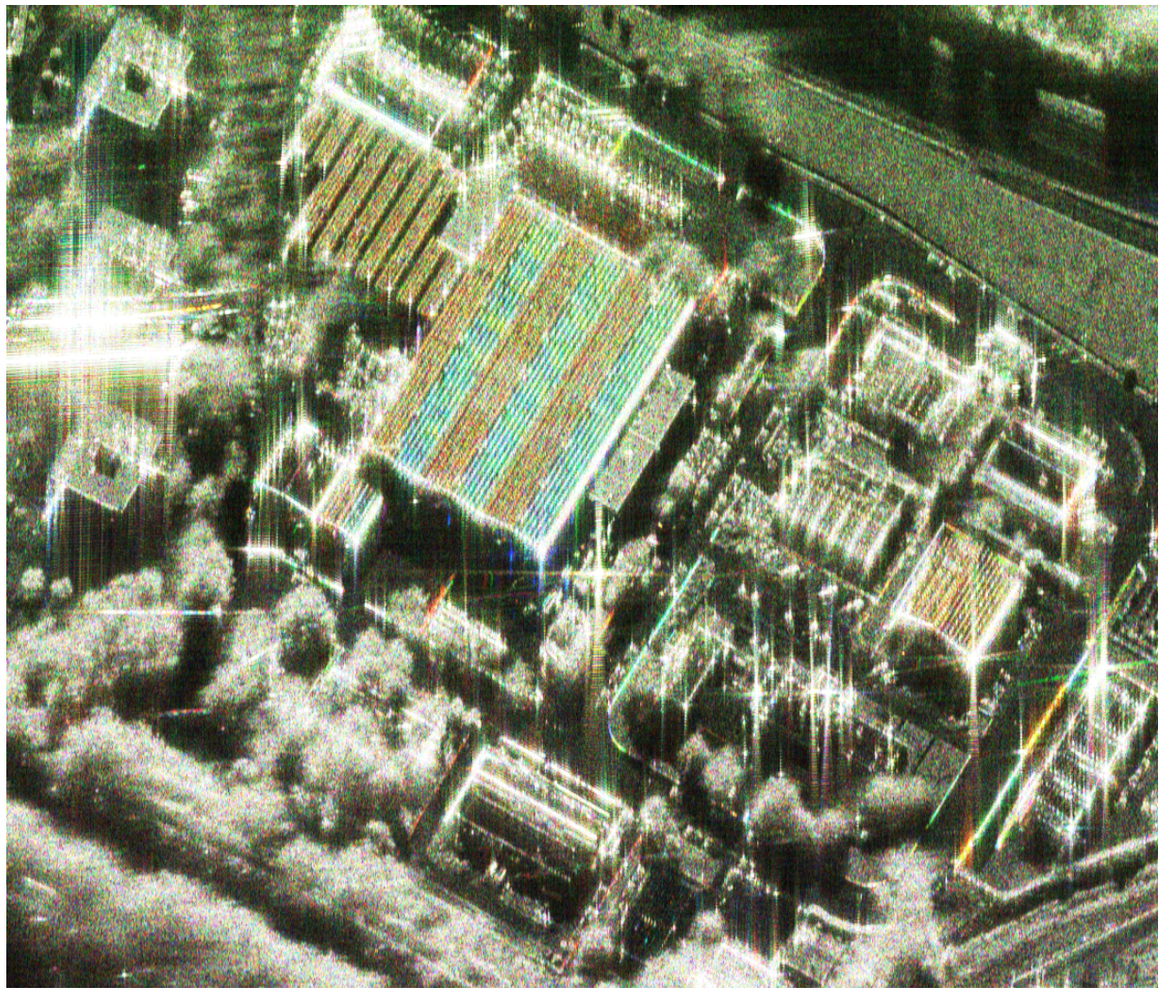
Examples: estimation of the polarimetric covariance matrix, interferometric coherency matrix

True Physical Behavior of Scatterers in SAR Imaging

elevation 30°



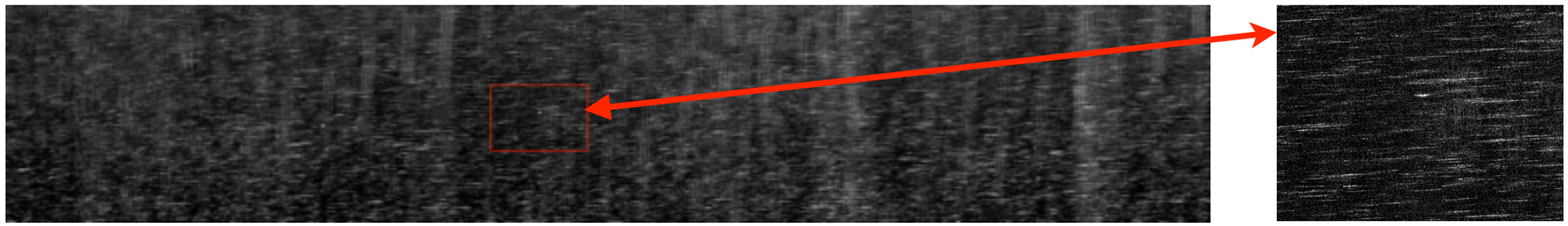
elevation 50°



Sub-band 1 **Sub-band 2** **Sub-band 3**

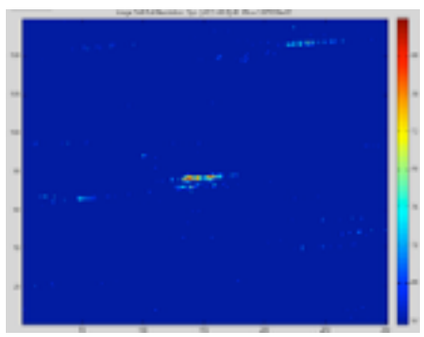
Scatterers have different behavior with regards to the frequency and direction of illumination: it means that this diversity can offer useful information to any detector exploiting it

Examples of Some Conventional Detection Schemes in Mono-Channel SAR Images



■ **Global thresholding (Gaussian hypothesis)**

$$\lambda = -\log(P_{fa})$$

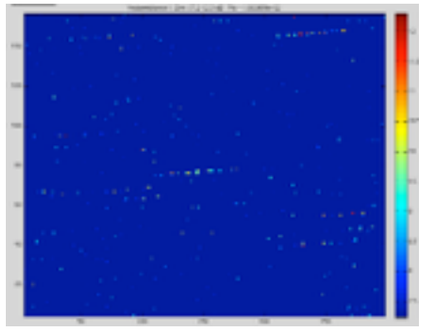


$$|x_i|^2 > \lambda$$

$$P_{fa} = 10^{-2}$$

■ **Local thresholding (Gaussian hypothesis)**

$$\lambda = N\left(P_{fa}^{-1/N} - 1\right)$$

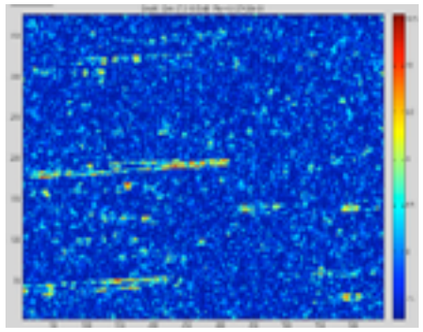


$$\frac{|x_i|^2}{\frac{1}{N}\sum_{k\neq i}^{N}|x_k|^2} > \lambda$$

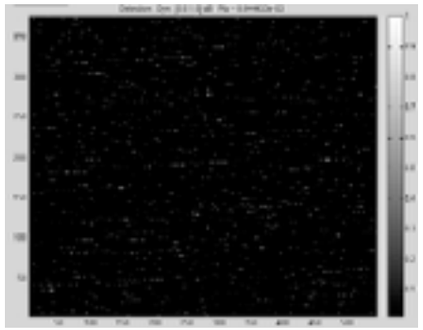
■ **Statistics-based thresholding (K-distribution)**

$$p_x(x) = \frac{2}{x\,\Gamma(\nu)\,\Gamma(L)}\left(\frac{L\,\nu\,x}{\mu}\right)^{(L+\nu)/2} K_{L-\nu}\left(2\sqrt{\frac{L\,x\,\nu}{\mu}}\right)$$

MLE estimation of parameters, determination of local threshold



local threshold



$$\frac{x}{\mu} > \lambda$$

Conventionnal SAR detection framework on a monodimensionnal SAR image mainly consists in locally thesholding the amplitude of the SAR image.

Examples of Some Conventional Detection and Classification Schemes in Multi-Channel SAR Images

Many conventional and well known SAR processing techniques such as:

- **Non Coherent Polarimetric Decomposition techniques used for classification,**
- **Speckle filtering techniques (Polarimetric Whitening Filter (PWF), Multilook PWF),**
- **Adaptive Detection schemes (Adaptive Matched Filter (AMF), Adaptive Kelly's Detector, Adaptive Normalized Matched Filter (ANMF), …),**
- **Change Detection schemes (statistical tests on the equality of covariance matrices)**

usually admit the multivariate zero-mean circular Gaussian statistic for spatial SAR pixels distribution:

$$p_g(\mathbf{k}) = \frac{1}{\pi^m \, |\mathbf{T}|} \, e^{-\mathbf{k}^H \, \mathbf{T}^{-1} \, \mathbf{k}}$$

and generally use Maximum Likelihood Estimate of the local covariance matrix *T* (coherency matrix), typically the Sample Covariance Matrix (SCM) built with *N* pixel-vectors k_i surrounding the pixel under test:

$$\hat{\mathbf{T}}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \, \mathbf{k}_i^H$$

leading to filters, detection tests with steering vector *p*:

$$\Lambda(\mathbf{k}) = \frac{\left|\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k}\right|^2}{\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

AMF

$$\Lambda(\mathbf{k}) = \frac{\left|\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k}\right|^2}{\left(\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{p}\right) \left(N + \mathbf{k}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Kelly

$$\Lambda(\mathbf{k}) = \frac{\left|\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k})\right|^2}{\left(\mathbf{p}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{p}\right) \left(\mathbf{k}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

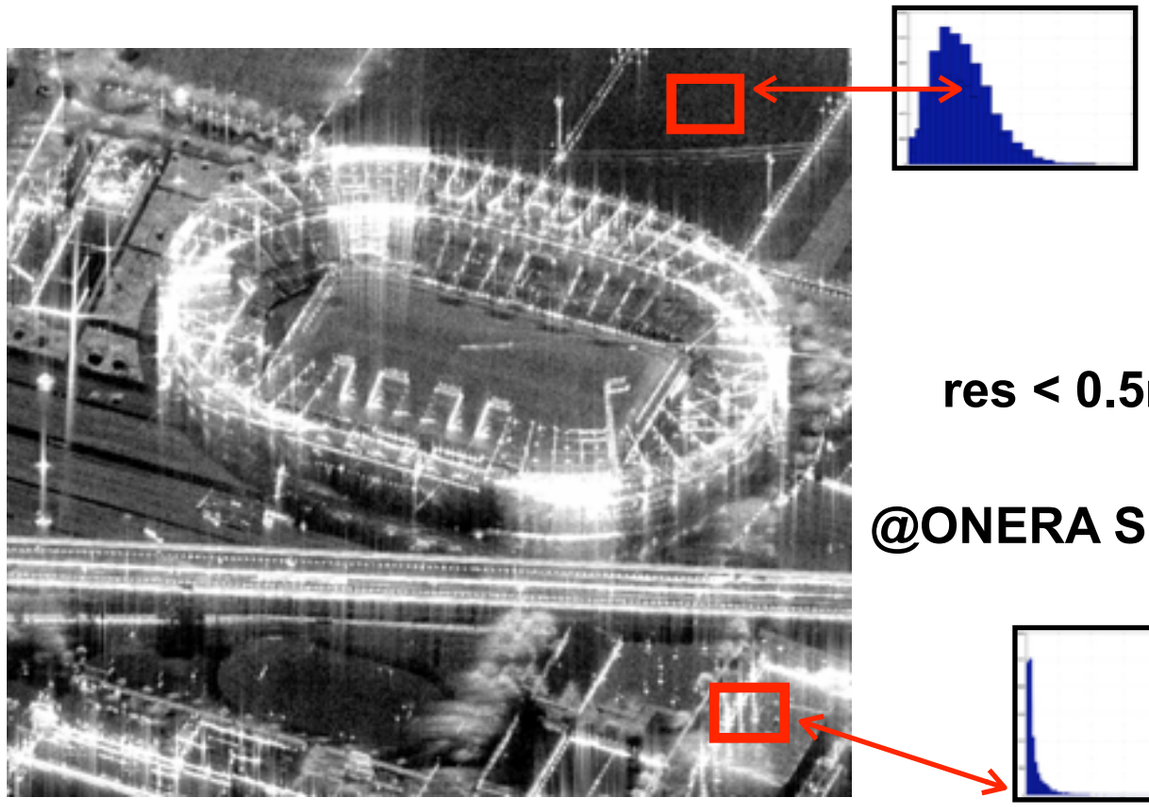
ANMF

$$\Lambda(\mathbf{k}) = \mathbf{k}^H \, \hat{\mathbf{T}}^{-1} \, \mathbf{k} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

PWF (Mahalanobis distance)

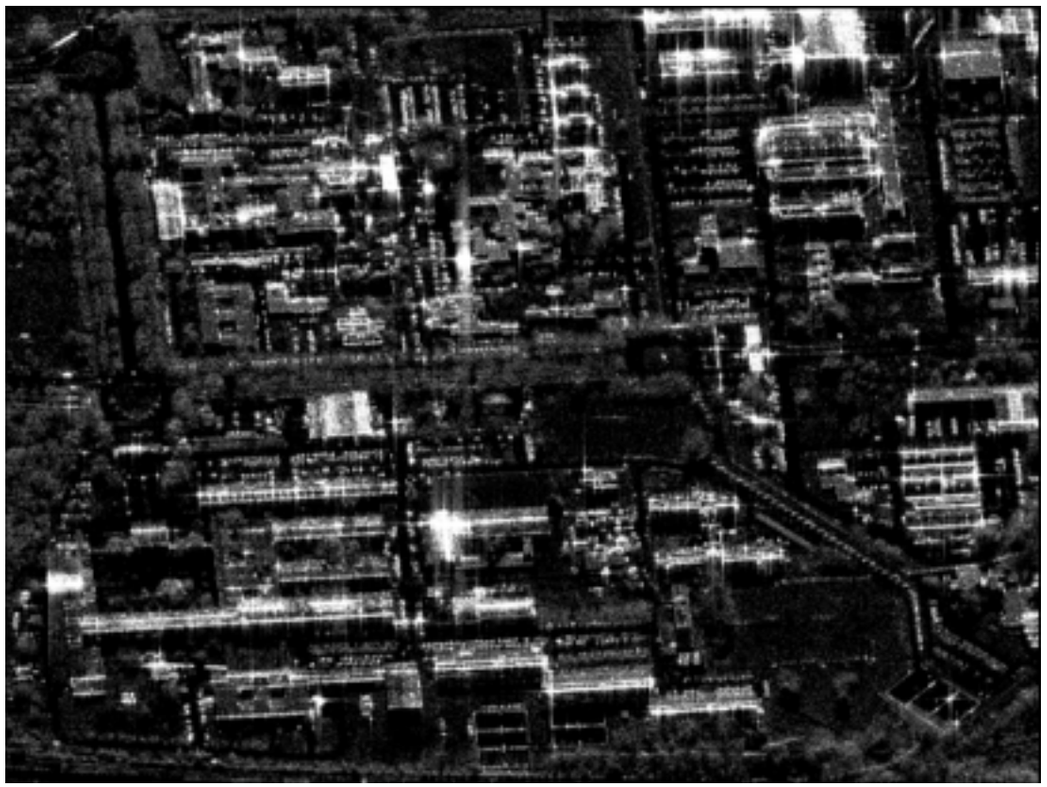
Challenging Problems Related to SAR Processing

- The SAR images are more and more complex, detailed, heterogeneous,
- The SAR pixels are colored and anisotropic,
- The spatial statistic of SAR images is not at all Gaussian !



res < 0.5m

@ONERA SETHI



- How to exploit, in adaptive detectors, the dispersive and anisotropic information of SAR pixels ?
- How to derive Multivariate Adaptive detectors (AMF, Kelly, ANMF) on a monodimensionnal SAR image without multi-channels such as polarimetry, interferometry, multi-passes SAR images ?
- How to enhance the performance of these Gaussian detectors in non-Gaussian environment ?

Time-Frequency Analysis for SAR Imaging

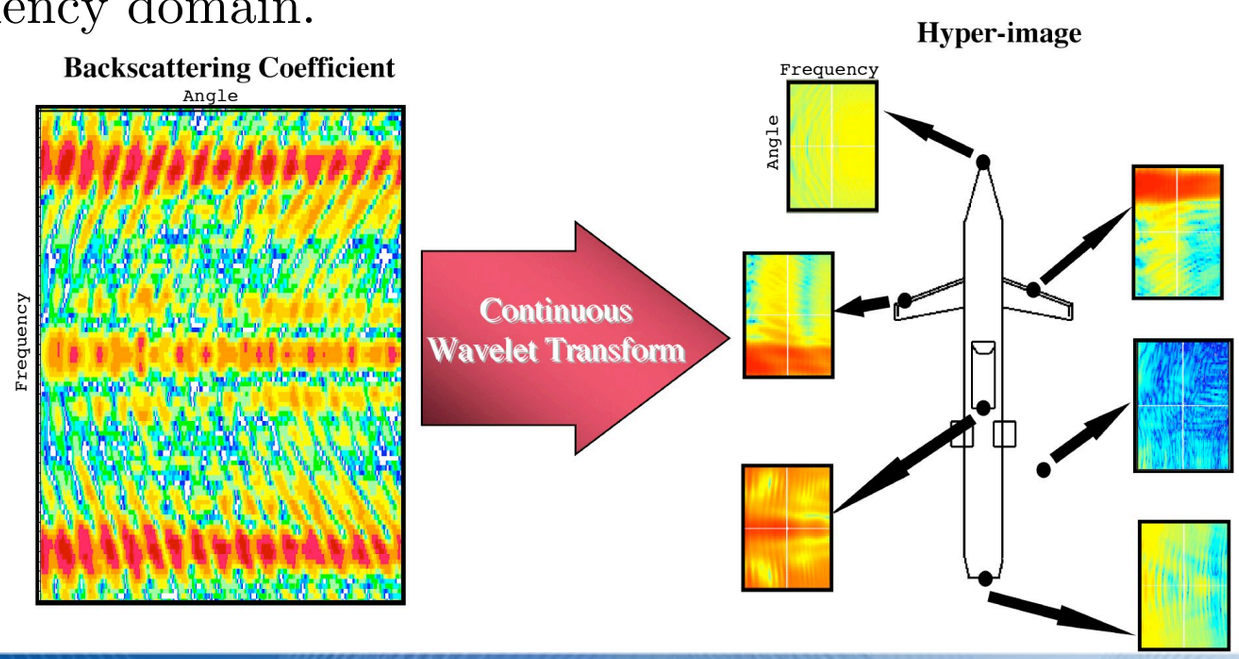
Time-Frequency Analysis allows to highlight the coloration and anisotropy properties of monodimensionnal SAR scatterers by characterizing each pixel of the SAR image with a vector of information related to angular or/and frequency behaviors [*Ovarlez et al.* 03].

The hyperimage is defined as a linear Time-Frequency decomposition of the backscattering coefficient $H(\mathbf{k})$:

$$A(\mathbf{x},\mathbf{k}) = \int_{\mathcal{D}_{\mathbf{u}}} H(\mathbf{u})\, \phi^H(\mathbf{u},\mathbf{k},\mathbf{x})\, e^{2\,i\,\pi\,\mathbf{u}^T\,\mathbf{x}}\, d\mathbf{u}$$

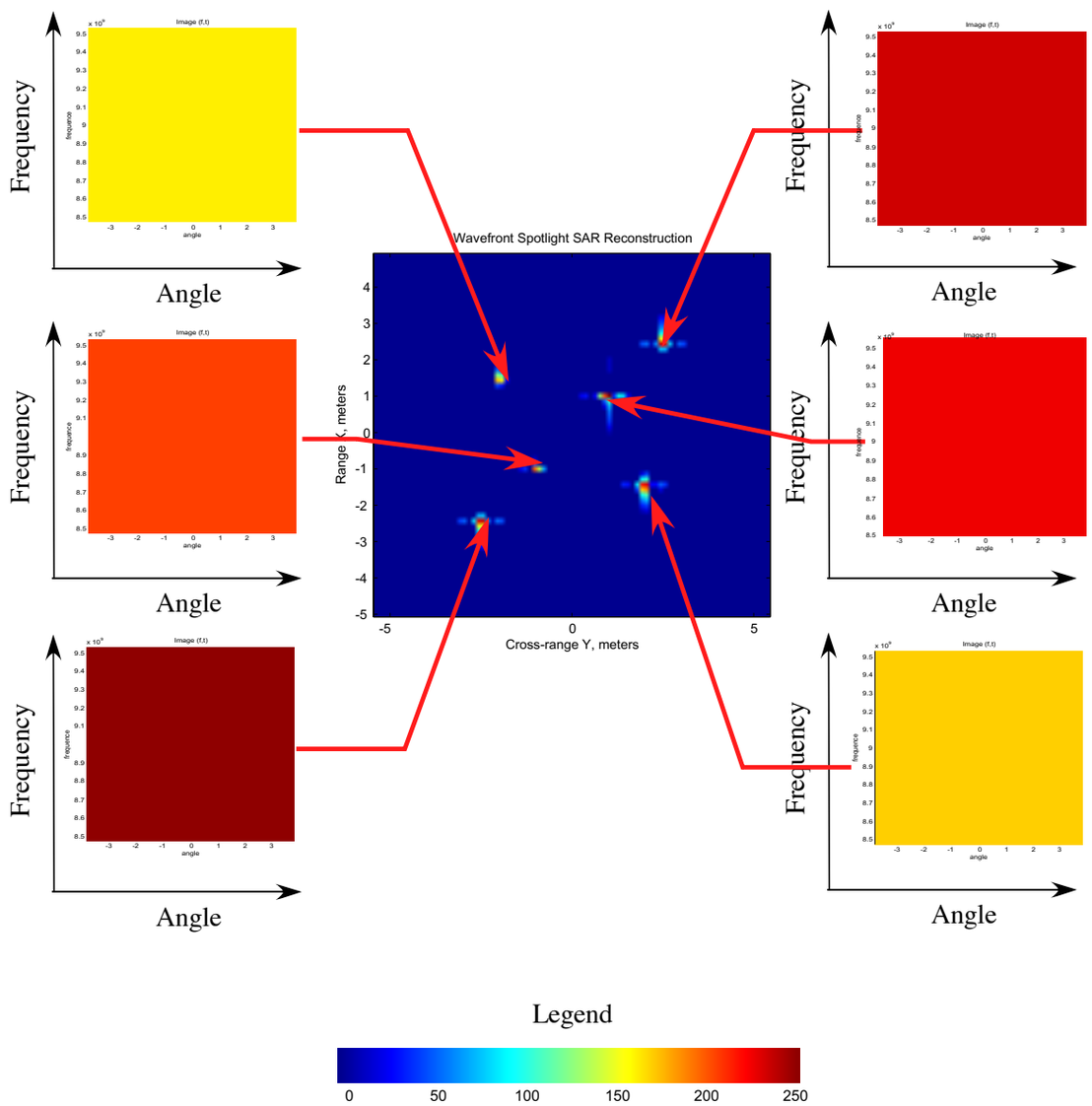
where $\phi(.)$ is an analyzing kernel acting on a mother wavelet $\phi_0(.)$ by given groups of transformation:

- group of translation in frequency domain and in angular domain: 2D short-time Fourier transform in angular and frequency domain,
- group of dilation in frequency domain and translation in angular domain : 2D wavelet transform in angular and frequency domain.

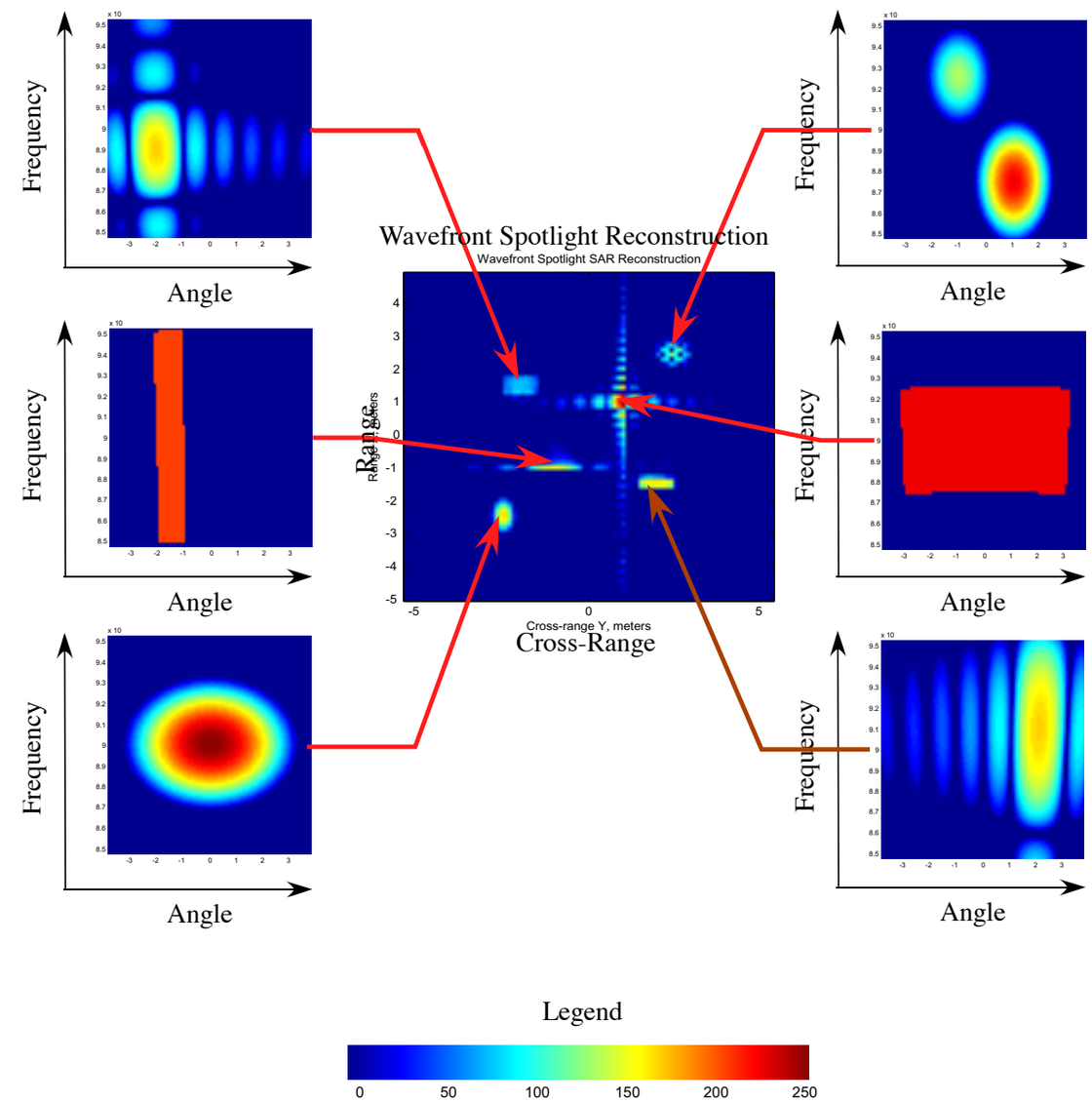


Comparison Between the Two Models

Example of theoretical model of isotropic and white scatterers



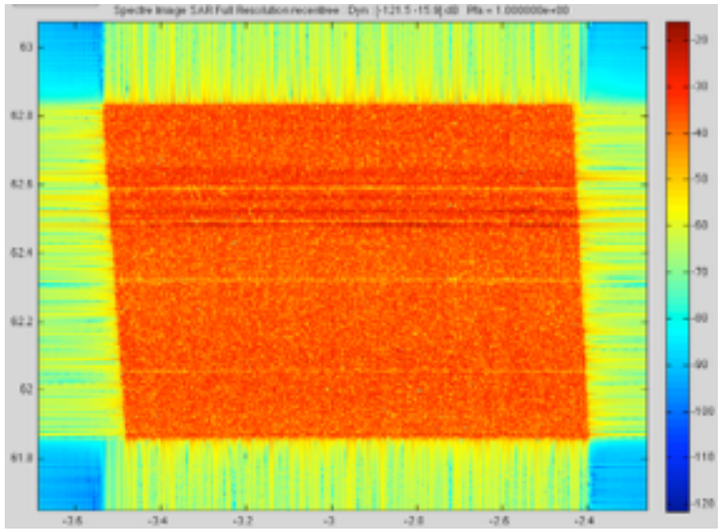
Example of theoretical model of anisotropic and colored scatterers

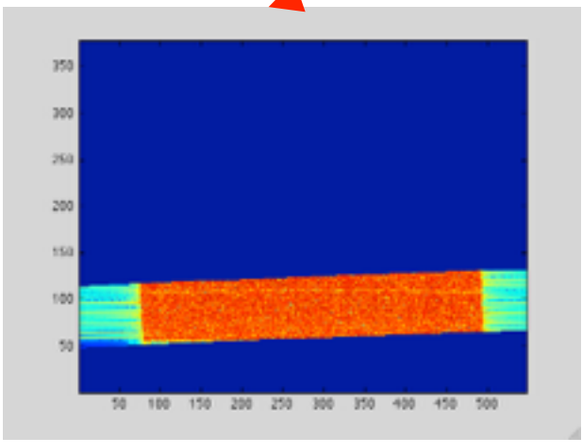


Decomposition of the SAR Backscatering Coefficient into Sub-bands and Sub-looks using Linear Time-Frequency Analysis

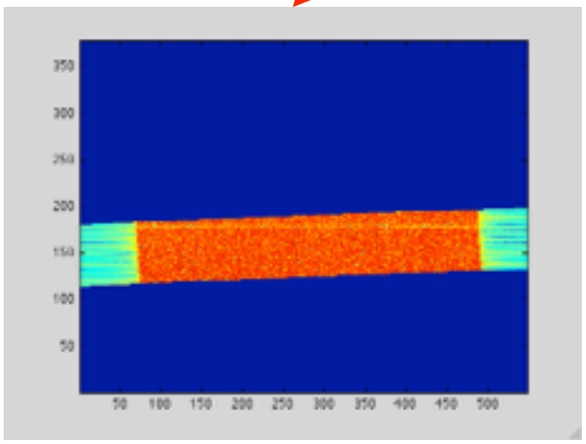
Backscattering coefficient $H(\mathbf{k})$

with $\mathbf{k} = \dfrac{2\,f}{c}\ (\cos\theta, \sin\theta)^T)$

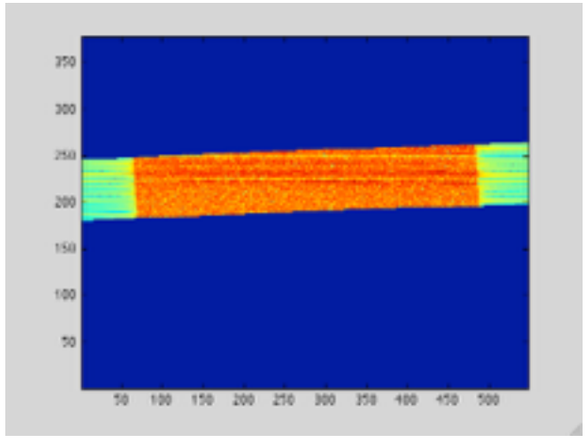




Sub-band 1

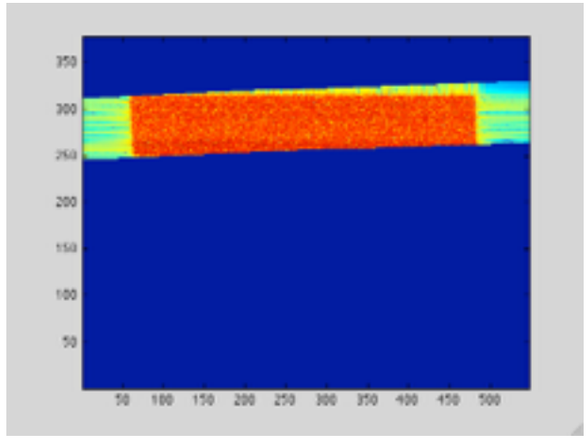


Sub-band 2



Sub-band 3

...

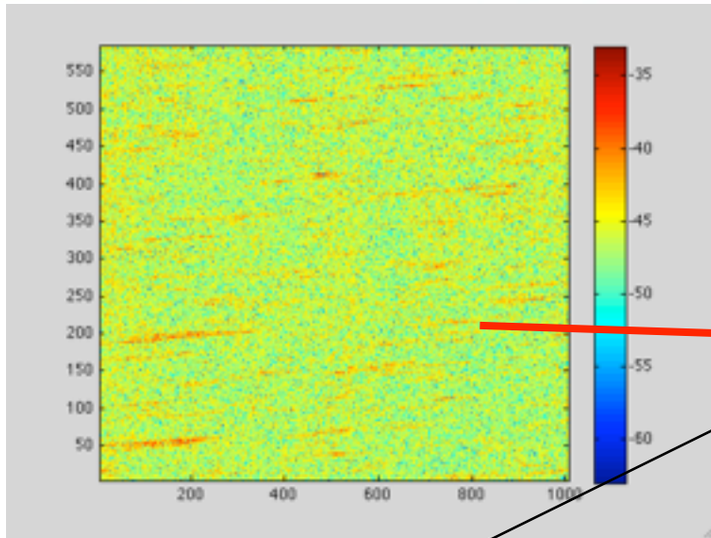


Sub-band N

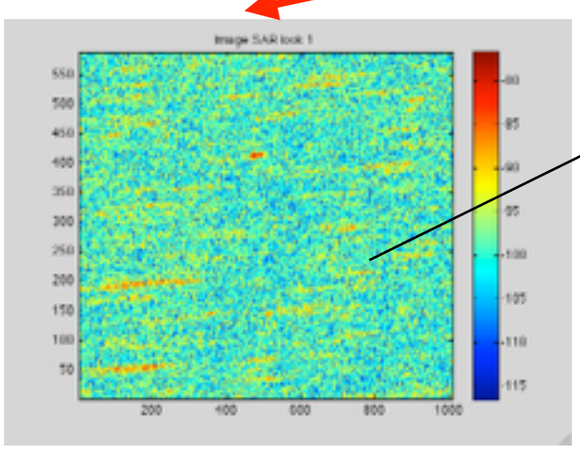
Decomposition of the SAR Image into sub-images using Linear Time-Frequency Analysis

High Resolution

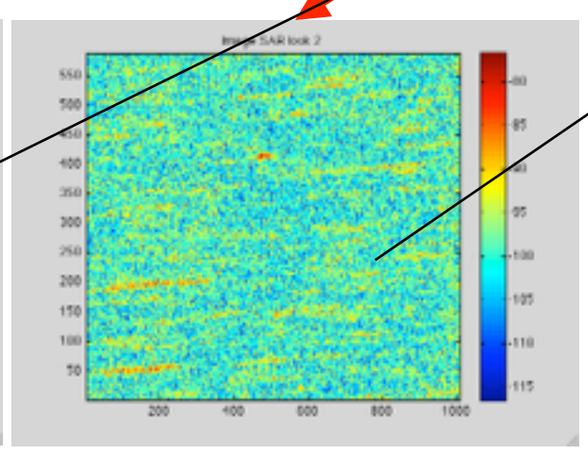
SAR Image



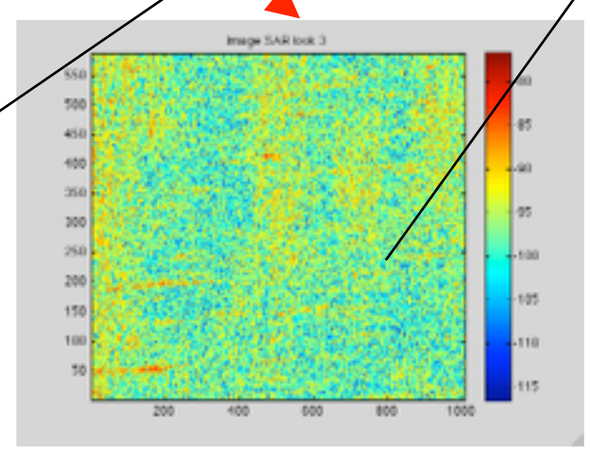
$$\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_N \end{pmatrix}$$



SAR Image Sub-image 1

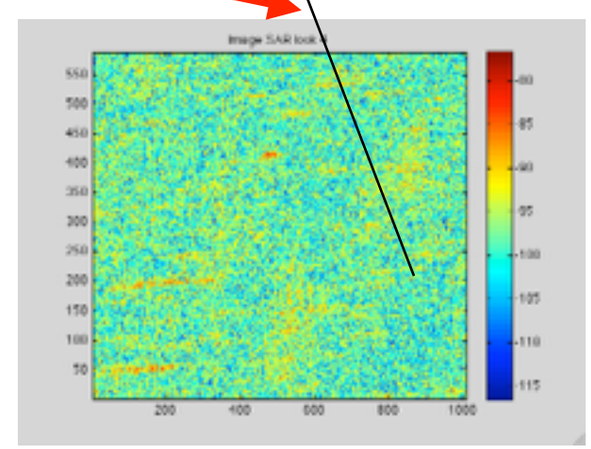


SAR Image Sub-image 2



SAR Image Sub-image 3

...



SAR Image Sub-images N

Each pixel characterizes now a N-vector of information related to coloration and anisotropy

Elliptical Distribution for SAR Background Modelling

Complex Elliptically Contoured Distributions [*Olilla 2003*]:

$$f_{\mathbf{c}}(\mathbf{c}) = |\mathbf{\Sigma}|^{-1}\, h_m\left(\mathbf{c}^H\, \mathbf{\Sigma}^{-1}\, \mathbf{c}\right)$$

- $\mathbf{c}$ **is a random complex *m*-vector characterizing each pixel of the SAR image,**
- $\sum$ **is the scatter matrix (equal to the covariance *T*, up to a scalar factor),**
- $h_m(.)$**, usually called density generator, is assumed to be known.**

Subclass of Spherically Invariant Random Vector: Compound Gaussian Process [*Yao 73*]

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{x}$$
$$\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$$
$$\tau \sim p(\tau)$$

$$h_m(u) = \int_0^{\infty} \frac{1}{(\pi\,\tau)^m} \exp\left(-\frac{u}{\tau}\right)\, p(\tau)\, d\tau$$

- **For a given set of spatial pixels of the SAR image, $\sum$ characterizes the correlation structure existing within the spectral bands or/and the azimutal bands (colored and anisotropic scatterers),**
- **Conditionally to the pixel, the m-vector is Gaussian. The texture variable τ characterizes the power variation of each vector from pixels to pixels (spatial heterogeneity).**

Powerful statistical model that allows:

- **to extend the Gaussian model (K, Weibull, Fisher, Cauchy, Alpha-Stable, Generalized Gaussian, etc.),**
- **to encompass the Gaussian model.**

Choice of the Covariance Matrix Estimators

Assuming *K* SIRV secondary data $\mathbf{c}_k = \sqrt{\tau_k}\,\mathbf{x}_k$ are available where $\mathbf{x}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$

- **The Sample Covariance Matrix SCM may be a «*poor*» estimator of the Elliptical/SIRV Scatter/Covariance Matrix Σ because of the texture contamination:**

$$\hat{\mathbf{M}}_{SCM} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{c}_k\,\mathbf{c}_k^H = \frac{1}{K}\sum_{k=1}^{K} \tau_k\,\mathbf{x}_k\,\mathbf{x}_k^H \neq \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_k\,\mathbf{x}_k^H$$

- **The Normalized Sample Covariance Matrix (NSCM) may be a good candidate of the Elliptical SIRV Scatter/Covariance Matrix Σ estimate:**

$$\hat{\mathbf{M}}_{NSCM} = \frac{m}{K}\sum_{k=1}^{K} \frac{\mathbf{c}_k\,\mathbf{c}_k^H}{\mathbf{c}_k^H\,\mathbf{c}_k} = \frac{m}{K}\sum_{k=1}^{K} \frac{\mathbf{x}_k\,\mathbf{x}_k^H}{\mathbf{x}_k^H\,\mathbf{x}_k}$$

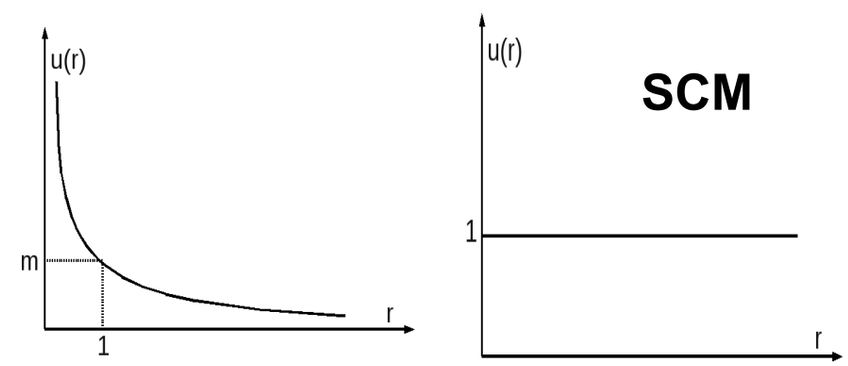
This estimate does not depend on the texture but it is biased and share the same eigenvectors but have different eigenvalues, with the same ordering) [*Bausson et al. 2006*].

Robust *M*-Estimators for SIRV and Elliptical Distributions

The complex *M*-estimators [*Huber 64, Maronna 76, Ollila 2012*] of location and scatter are defined as the joint solutions of:

$$\hat{\boldsymbol{\mu}} = \frac{\sum_{n=1}^{K} u_1(t_n)\mathbf{c}_n}{\sum_{n=1}^{K} u_1(t_n)}, \qquad \hat{\mathbf{M}} = \frac{1}{K}\sum_{n=1}^{K} u_2(t_n^2)(\mathbf{c}_n - \hat{\boldsymbol{\mu}})(\mathbf{c}_n - \hat{\boldsymbol{\mu}})^H$$

where $t_n = \left((\mathbf{c}_n - \hat{\boldsymbol{\mu}})^H \hat{\mathbf{M}}^{-1}(\mathbf{c}_n - \hat{\boldsymbol{\mu}})\right)^{1/2}$.



$u_2(.)$ **choice**

- u_1, u_2 **are two weighting functions acting on the quadratic form, i.e. *Mahalanobis* distance,**
- **The choice of** u_1, u_2 **results in different estimates for the covariance matrix and the mean vector,**
- **Existence and uniqueness of the solution have been proven provided** u_1, u_2 **satisfy given conditions** [*Maronna 1976*],
- **Consistency and Asymptotical Gaussian distribution of these estimates [*Mahot et al. 2012*]**
- **Robust to outliers, to the presence of strong targets or high impulsive samples in the *K* reference cells,**
- **Generalization of Maximum Likelihood Estimators:**

$$u_2\left(t^2\right) = u_1(t) = -h'_m\left(t^2\right)/h_m\left(t^2\right)$$

A Particular M-Estimate: The Tyler's Estimate

For an unknown but deterministic texture parameter, the Maximum Likelihood Fixed Point estimate of the Covariance is found [*Tyler 87, Conte-Gini 02*] to be the solution of the following implicit equation:

Fixed Point (FP)

$$\hat{\mathbf{M}}_{FP} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \hat{\mathbf{M}}_{FP}^{-1} \, \mathbf{c}_k} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{x}_k \, \mathbf{x}_k^H}{\mathbf{x}_k^H \, \hat{\mathbf{M}}_{FP}^{-1} \, \mathbf{x}_k}$$

This estimate is an approached MLE in the general SIRV context and is called the Tyler's M-estimate.

[*F. Pascal et al. 2006*]

- **This estimate is independent on the texture parameter,**
- **Consistent, unbiased, robust, asymptotically Gaussian estimate and supposed to be, at a fixed number *K*, Wishart distributed with *mK/(m+1)* degrees of freedom,**
- **Existence and unicity of the solution are proven. The solution can be reached by recurrence M_k=f(M_{k-1}) whatever the starting point M_0 (ex: M_0=*I*, M_1=M_{NSCM}),**

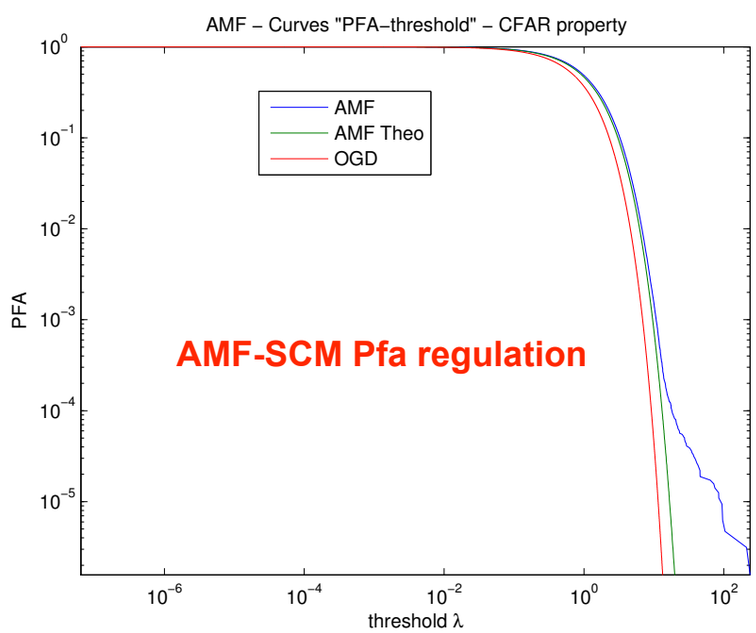
Robust CFAR Detector for SAR Imaging

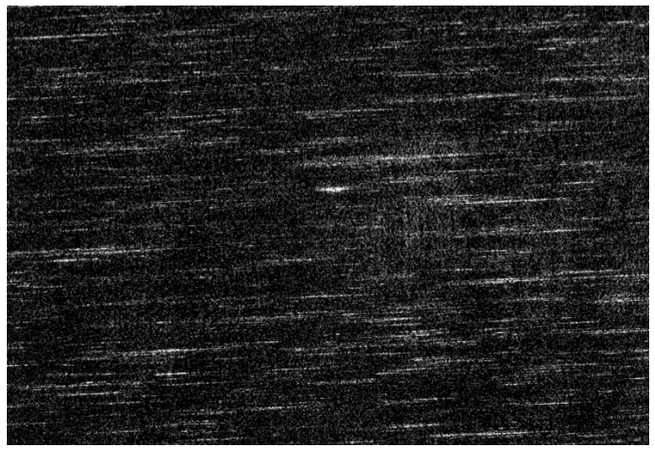
New detectors called Robust Adaptive Detectors can be derived by replacing in the Normalized Matched Filter any « robust *estimate* » of the covariance matrix.

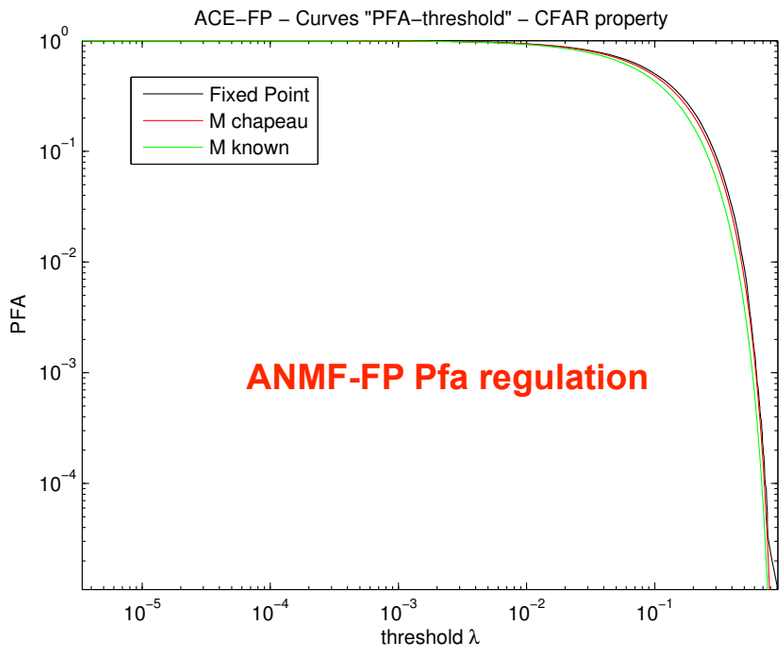
ACE : Adaptive Cosine Estimator
ANMF : Adaptive Normalized Matched Filter

$$\Lambda(\mathbf{c}) = \frac{|\mathbf{p}^H \hat{\mathbf{M}}^{-1} \mathbf{c}|^2}{\left(\mathbf{p}^H \hat{\mathbf{M}}^{-1}\mathbf{p}\right)\left(\mathbf{c}^H \hat{\mathbf{M}}^{-1} \mathbf{c}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda.$$

These detectors, homogeneous of degree 0, are SIRV-CFAR only for some particular estimates of Σ : the test statistic does not depend at all on the texture PDF and on the unknown scatter matrix Σ







These detectors can therefore perfectly regulate the False Alarm Rate whatever the spatial heterogeneity of the SAR image

Conclusions

- **Time-Frequency Analysis can help in caracterizing spectral and anisotropic behavior of each pixel in monodimensional SAR image (with phase),**

- **The SIRV and Elliptically Contoured Distribution clutter model allows to take into account the clutter complexity:**
  - **spatial non-Gaussianity or/and heterogeneity (spatial clutter power fluctuations) of the SAR image,**
  - **spectral and anisotropic behavior of the clutter,**

- **By linking these two models together, the robust ANMF detector built with the Fixed Point estimator (or any M-estimator)**
  - **is shown to be CFAR-texture, CFAR-matrix,**
  - **exhibits nice properties (robustness), good regulation of the Pfa in highly textured SAR image leading to very good detection performance.**