New Doppler Processing for the Detection of Small and Slowly-Moving Targets in Highly Ambiguous Radar Context

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Abstract— **In this paper the problem of target detection and localization in the context of pulse Doppler radars suffering Doppler ambiguities is analyzed. A new multiple-burst method based on the Iterative Adaptive Approach (IAA) is proposed to resolve ambiguities from multiple burst signals with Pulse Repetition Frequency (PRF) and carrier frequency diversity.**

Keywords— **Pulse Doppler radars, Doppler ambiguities, IAA.**

I. INTRODUCTION

Pulse Doppler radars operating with an intermediate or a low PRF suffer from Doppler ambiguities. Ambiguities resolution problem is generally addressed by transmitting successive pulse trains with different repetition frequencies, producing different unambiguous velocities. One limitation of this technique concerns the poor Doppler resolution resulting from the shorter coherent duration allowed to each pulse train.

Conventional Doppler estimation methods with good resolution capabilities like Capon's method [1] and MUSIC [2] can not be applied to resolve ambiguities from burst signals with PRF diversity where only a single observation is available per PRF value, they require the estimation of the covariance matrix from several observations. The well known MaxiIAAmum Likelihood (ML) approach [3] is based on a criterion that can handle differently sampled burst signals and resolve ambiguities, however, its implementation can be extremely complex since it requires the determination of the problem order i.e. number of targets and the resolution of a nonlinear problem. The Iterative Adaptive Approach [4] is an iterative solution to the Deterministic Maximum Likelihood (DML) criterion. In [5], an improved IAA algorithm for ambiguities resolution in multiple PRF radars has been proposed, but it is not adapted to frequency agile systems were target echoes are varying from a burst to another. In this paper, IAA is revised to deal with the problem of Doppler ambiguities resolution from burst signals with different PRFs and carrier frequencies. Pulse Doppler radars operating with an intermediate or a low PRF suffer from Doppler ambiguities. Ambiguities resolution problem is generally addressed by transmitting successive pulse trains with different repetition frequencies, producing different unambiguous velocities. One limitation of this technique concerns the poor Doppler resolution resulting from the shorter coherent duration allowed to each pulse train.

The remainder of this paper is organized as follows. Section 2 introduces the signal model. Then the proposed approach is described in Section 3. It is evaluated and compared to IAA in section 4 and conclusions are drawn in Section 5.

II. SIGNAL MODEL

This paper considers a ground-based pulse Doppler radar operating at an intermediate PRF with a maximum unambiguous velocity $v_{amb} \simeq 200 m/s$ with c the speed of the light and f_c the carrier frequency. In practice, target radial velocities may vary from $5m/s$ (domestic aerial unnamed vehicle) to $2000m/s$ (fighter aircraft). The Doppler ambiguity factor $|\frac{v_i}{v_{amb}}|$ is therefore much greater than one, it reach values up to 10. For the sake of simplicity, only the Doppler axes is considered, targets are supposed to be unambiguous in range. To resolve Doppler ambiguities, successive coherent bursts with PRF diversity are transmitted. Each burst consists of uniformly spaced pulses whereas the number of pulses M_k and the pulse repetition frequency PRF_k are different from one burst to the other. The carrier frequency is also generally varied from burst to burst in order to enhance detection performance by forcing target echo decorelation between the different bursts. Considering N_t targets within the radar scene, the received signal is generally formed by the sum of their responses in addition to undesired acquisition noise $\mathbf{n}$. After range-compression and by assuming no range migration all through duration of successive bursts, the received signal from the M_k pulses of the k^{th} burst in a single range-cell can be expressed in the following form

$$\mathbf{y}_k = \sum_{i=1}^{N_t} x_{k,i} \mathbf{a}_k (v_i) + \mathbf{n}_k \ \in \mathbb{C}^{M_k} \tag{1}$$

Where,

> $x_{k,i}$ is the i^{th} target response complex amplitude. Burst-wise frequency diversity forces target echo decorrelation. Under the hypotheses that the burst duration is small, $x_{k,i}$ is supposed to be constant during each burst and varies from one burst to another as a stationary zero-mean Gaussian process.

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$\mathbf{a}_k$ denotes the target Doppler signature, or the Doppler vector corresponding to the velocity v_i, $\mathbf{a}_k(v_i) = [1, \dots, e^{j2\pi(M-1)\frac{v_i}{v_{amb_k}}}]$.
$\mathbf{n}_k$ is a Gaussian white noise.

III. DOPPLER AMBIGUITY RESOLUTION WITH MULTIPLE-BURST IAA

The Iterative Adaptive Approach proposed in [4] is an iterative spectral estimation method based on Carathodory model of the covariance matrix [6]. The original version of IAA was developed to manage burst signals with identical PRFs. In this section, it is revised to deal with burst signals with PRF diversity in order to resolve Doppler ambiguities.
The proposed Multiple-Burst IAA (MBIAA) algorithm estimates the Doppler map of complex reflectivies $\{x_{k,i}\}_{i=1}^{N_v}$, where N_v is the number of velocities forming a grid of potential target radial velocities $\mathbf{v} = [v_1, \dots, v_{N_v}]$, by minimizing for each radial velocity v_i, the following APES-like [7] criterion

$$\min_{x_{k,i}} \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{a}_k(v_i)\, x_{k,i}\|^2_{\mathbf{Q}_k} \tag{2}$$

with

$$\mathbf{Q}_k = \mathbf{R}_k - |x_{k,i}|^2 \mathbf{a}_k(v_i)\, \mathbf{a}_k^H(v_i) \tag{3}$$

where $\|\mathbf{s}\|^2_{\mathbf{Q}_k} \triangleq \mathbf{s}^H \mathbf{Q}_k^{-1} \mathbf{s}$.
The solution of this problem is given by [4]

$$\widehat{x}_{k,i} = \widehat{x}_k(v_i) = \frac{\mathbf{a}_k^H(v_i)\, \mathbf{Q_k}^{-1} \mathbf{y}_k}{\mathbf{a}_k^H(v_i)\, \mathbf{Q}_k^{-1} \mathbf{a}_k(v_i)} = \frac{\mathbf{a}_k^H(v_i)\, \mathbf{R}_k^{-1} \mathbf{y}_k}{\mathbf{a}_k^H(v_i)\, \mathbf{R}_k^{-1} \mathbf{a}_k(v_i)} \tag{4}$$

This solution require the estimation of the unknown covariance matrix $\mathbf{R}_k$. According to the classical IAA algorithm the covariance matrix $\widehat{R}_k$ of the k^{th} burst is estimated as follows

$$\widehat{\mathbf{R}}_{\mathbf{k}} = \mathbf{A}_k(\mathbf{v})\, \mathrm{diag}(\mathbf{b})\, \mathbf{A}_k^H(\mathbf{v}) \tag{5}$$

where $[\mathbf{b}]_i = |x_{k,i}|^2$. A first estimate of $x_{k,i}$ is given by matched filtering

$$\widehat{x}^0_{k,i} = \frac{\mathbf{a}_k^H(v_i)\, \mathbf{y}_k}{\mathbf{a}_k^H(v_i)\, \mathbf{a}_k(v_i)} \tag{6}$$

In the proposed MBIAA algorithm the differently sampled burst signal covariance matrices are interpolated in the spectral domain to contribute all together to the estimation of each burst covariance matrix. The multiple-burst covariance matrix $\widehat{R}_{MB_k}$ of the k^{th} burst is estimated from the signals of a the different bursts as follows

$$\widehat{R}_{MB_k} = \frac{1}{K} \sum_{k'=1}^{K} \boldsymbol{\beta}_{k' \to k} \widehat{\mathbf{R}}_{\mathbf{k'}} \boldsymbol{\beta}^H_{k' \to k} \tag{7}$$

with

$$\boldsymbol{\beta}_{k' \to k} = \mathbf{A}_k \tilde{\mathbf{A}}^H_{k' \to k} \tag{8}$$

where $\mathbf{A}_k \in \mathbb{C}^{M_k \times M_k}$ is the inverse Discrete Fourier Transform (DFT) and $\tilde{\mathbf{A}}^H_{k' \to k} \in \mathbb{C}^{M_{k'} \times M_k}$ is the transformation matrix that interpolates the elements $\widehat{\mathbf{R}}_{\mathbf{k'}}$ into the spectral coordinates of the k^{th} burst

$$\tilde{\mathbf{A}}_{k' \to k} = \frac{1}{\sqrt{M_{k'}}} \begin{pmatrix} 1 & \cdots & 1 \\ 1 & \cdots & e^{j2\pi \frac{(M_k-1) f_{c_{k'}}}{f_{c_k} PRF_{k'}} \delta f_{d_k}} \\ \vdots & & \vdots \\ 1 & \cdots & e^{j2\pi M_{k'} \frac{(M_k-1) f_{c_{k'}}}{f_{c_k} PRF_{k'}} \delta f_{d_k}} \end{pmatrix} \tag{9}$$

where $\delta f_{d_k} = \frac{PRF_k}{M_k}$ denotes the Doppler resolution.
It can be noted that for $k' = k$, $\boldsymbol{\beta}_{k' \to k} = \mathbf{I}_{M_k}$, where $\mathbf{I}_\alpha$ is the identity matrix of size α. As summarized in algorithm 1, the MBIAA alternates the estimation of $\mathbf{R}_{MB_k}$ and $\mathbf{x}_k$ until convergence, this means after a maximum number of iterations is achieved or after the stabilization of the solution. The algorithm refines progressively the initial estimates provided by the matched filter over an APES-like filtering. As the classical version of IAA the MBIAA estimates, at each iteration, the covariance matrix from the diagonal matrix formed by the previous estimates of the signal power at the grid velocities making it able to work even with a small number of observations.

Algorithm 1 MBIAA

```
Initialization: Matched filter
v = [v_1, ..., v_{N_v}]
For $i = 1 : N_v$
    For $k = 1 : K$
        $\widehat{x}_k(v_i) = \frac{\mathbf{a}_k^H(v_i)\mathbf{y}_k}{\mathbf{a}_k^H(v_i)\mathbf{a}_k(v_i)}$
    $[\mathbf{b}]_i = |\widehat{x}_k(v_i)|^2$
Do
    $\widehat{R}_{MB_k} = \frac{1}{K}\sum_{k'=1}^{K} \boldsymbol{\beta}_{k'\to k}\mathbf{A}_k(\mathbf{v})\,\mathrm{diag}(\mathbf{b})\,\mathbf{A}_k^H(\mathbf{v})\,\boldsymbol{\beta}^H_{k'\to k}$

    For $i = 1 : N_v$
        For $k = 1 : K$
            $\widehat{x}_k(v_i) = \frac{\mathbf{a}_k^H(v_i)\widehat{\mathbf{R}}^{-1}_{\mathbf{MB_k}}\mathbf{y}_k}{\mathbf{a}_k^H(v_i)\widehat{\mathbf{R}}^{-1}_{\mathbf{MB_k}}\mathbf{a}_k(v_i)}$
        $[\mathbf{b}]_i = |\widehat{x}_k(v_i)|^2$
Until convergence
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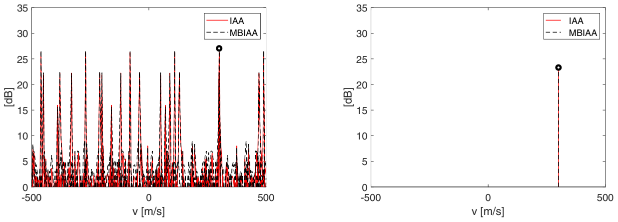
In order to detect targets and resolve ambiguities, the values corresponding to the peaks of $\mathbf{x}_k$,$k = 1 : K$ are compared to a detection threshold and only the peaks whose values exceed the threshold at least N $(1 < N < K)$ times are detected, this is known as N from K detection. In general, at least $N = 3$ burst detections are required to solve ambiguities [8].

IV. PERFORMANCE ASSESSMENT

In this section, the performance of the proposed MBIAA algorithm are evaluated and compared to the ones of IAA. Five burst signals with PRF and frequency diversity are considered. Signal characteristics are summarized in table 1, all bursts have the same velocity resolution δv but different from each other in terms of their carrier frequency and PRF. It is assumed that each signal contains a Gaussian white noise of $0dB$.

Table 1. Observed signal characteristics

Parameters	burst 1	burst 2	burst 3	burst 4	burst 5
f_c (GHz)	3	3.01	3.025	3.015	3.02
PRF (Hz)	3400	3812.7	4235	4623	5033.3
M	17	19	21	23	25
δv (m/s)	10	10	10	10	10
v_{amb} (m/s)	170	190	210	230	250



(a) (b)

Fig. 1. Five bursts superposed IAA and MBIAA estimates: (a) Before detection; (b) After N from K detection.

Figure 1 shows the result of IAA and MBIAA combined to the N from K detection test on a scenario with an ambiguous target, $v = 200m/s$. The results of IAA and MBIAA on all bursts before detection are superposed in figure 1a. In both cases, many peaks with the same intensity as the target appear in addition to the one located at the real target velocity, this means that the two algorithms are sensitive to ambiguities. After applying the 3 from 5 detection test, in figure 1b, only one response is detected and the target were correctly localized.

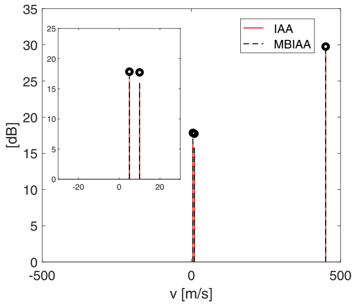


Fig. 2. IAA and MBIAA combined to the N from K detection test 'multiple-targets scenario'.

Figure 2 shows the result of MBIAA and IAA in a multiple-targets scenario including three targets. The proposed scenario contains two unambiguous and closely spaced targets $v_1 = 5m/s$, $v_2 = 10m/s$ ($|v_1 - v_2| = \frac{\delta v}{2}$) and an isolated ambiguous target $v_3 = 450m/s$. The two algorithms were able to separate the closely spaced targets. The three targets have been detected and unambiguously localized at their true radial velocity.

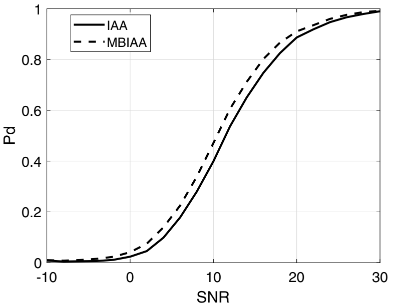


Fig. 3. Probability of detection for different SNRs. $P_{fa} = 10^{-4}$.

In figure 3, the detection performance of IAA and MBIAA are evaluated. The curves result from 500 Monte-Carlo runs where the Gaussian white noise varies in each trial. They represent the probability to detect an unambiguous target from a signal with the characteristics of burst 1 in table 1 with respect to the SNR. The detection threshold correspond to a P_{fa} of 10^{-4}. In MBIAA, the covariance matrix is jointly estimated from the five bursts. It can be seen that MBIAA outperforms IAA in term of detection performance, a gain of about $1dB$ is obtained for a P_d of 0.5.

V. CONCLUSION

In this paper a new multiple-bursts algorithm for Doppler ambiguities resolution has been proposed. The MBIAA algorithm associate different burst signals with PRF and frequency diversity in order to estimate the Doppler map of each burst. Combined to the N from K detection test, MBIAA resolves the Doppler ambiguities. Simulation results showed that the proposed approach enhance detection performance with respect to IAA.

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