ROBUST DETECTION USING THE SIRV/ ELLIPTICAL BACKGROUND MODELLING FOR HYPERSPECTRAL IMAGING

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OUTLINE OF THE TALK

- Problems Description and Motivation,
- The Spherically Invariant Random Process and Elliptical Process Modelling for Hyperspectral Imaging,
- Estimation in the SIRV/Elliptical Background,
- Detection in the SIRV/Elliptical Background,
- Anomaly Detection and Target Detection Results on Experimental Data,
- Conclusion.

PROBLEMS DESCRIPTION

- **ANOMALY DETECTION IN HYPERSPECTRAL IMAGES**

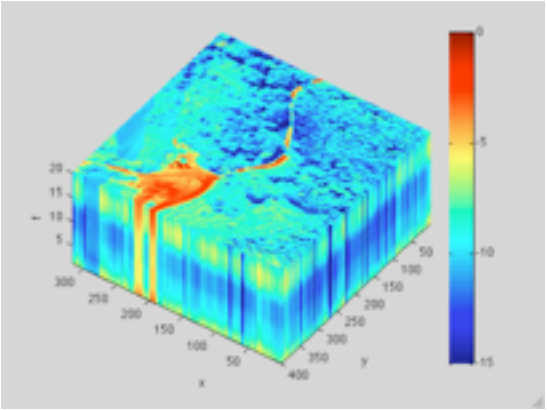
  To detect all kind of targets that are « different » from the background - Regulation of False Alarm (Mahalanobis Distance).

- **DETECTION OF TARGETS IN HYPERSPECTRAL IMAGES**

  To detect (GLRT) targets characterized by a given spectral signature - Regulation of False Alarm.

- **CLASSIFICATION IN HYPERSPECTRAL IMAGES**

  Put the background in separate homogeneous classes.



Example of DSO Hyperspectral Image (2010)

CONVENTIONAL METHODS OF DETECTION

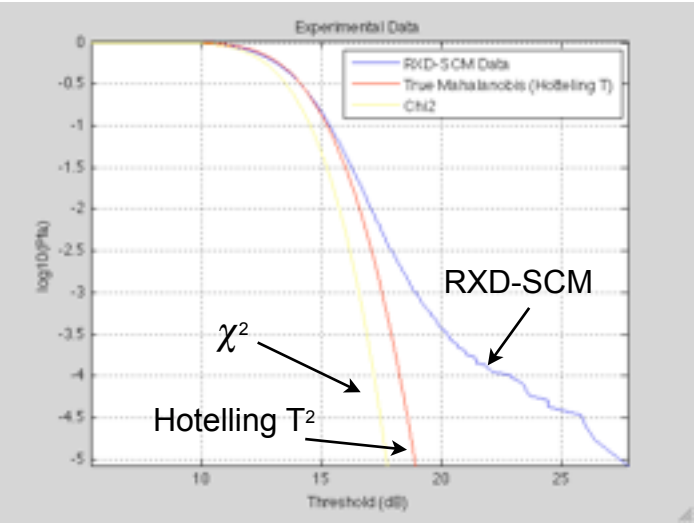
- Many methodologies for detection and classification in hyperspectral images can be found in radar detection community. We can retrieve all the detectors family commonly used in radar detection (AMF (intensity detector), ACE (angle detector), sub-spaces detectors, ...).

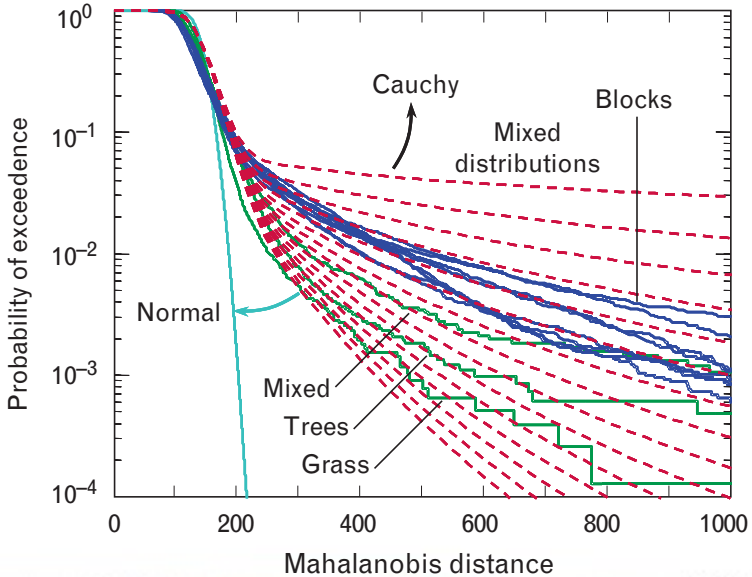
- **Almost all** the conventional techniques for anomaly detection and targets detection are based on Gaussian assumption and on spatial homogeneity in hyperspectral images.

BUT .....

- Hyperspectral data are **spatially heterogeneous** in intensity and/or **cannot be only characterized by Gaussian statistic**:

DSO Data





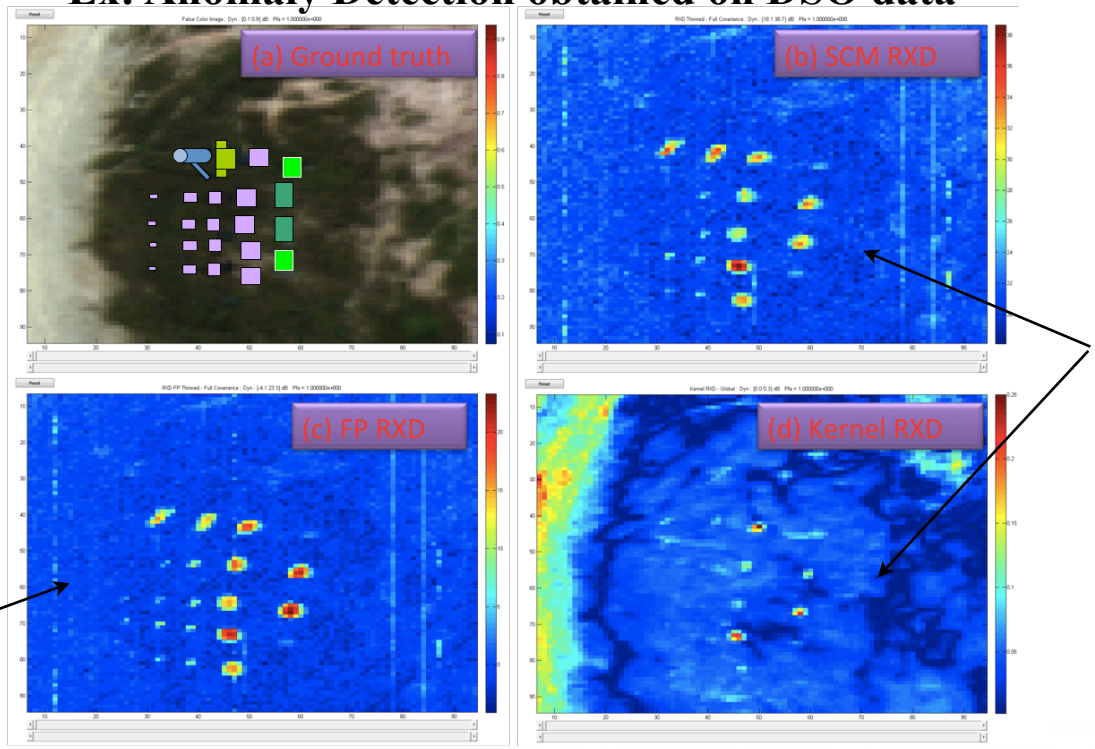
[*Manolakis 2002*]

SOME COMMENTS

- Spherically Invariant Random Vectors (SIRV) models have been started to be studied in the Hyperspectral scientific community **but everybody still uses .... Gaussian estimates** !

- **SIRV models with good appropriate estimates** have been found to be efficient for false alarm regulation as well as for detection schemes in non-Gaussian and heterogeneous background for Hyperspectral Imaging (*2010 J.P. Ovarlez DSO attachment, IEEE IGARSS 2011*).

Ex: Anomaly Detection obtained on DSO data



Classical results

NEW RESULT

SONDRA/ONERA BACKGROUND FOR HYPERSPECTRAL IMAGING

All the previous signal processing methodologies conducted in SONDRA/ ONERA since long years for radar and SAR processing can be "easily" and "naturally" extended and exploited for Hyperspectral Imaging:

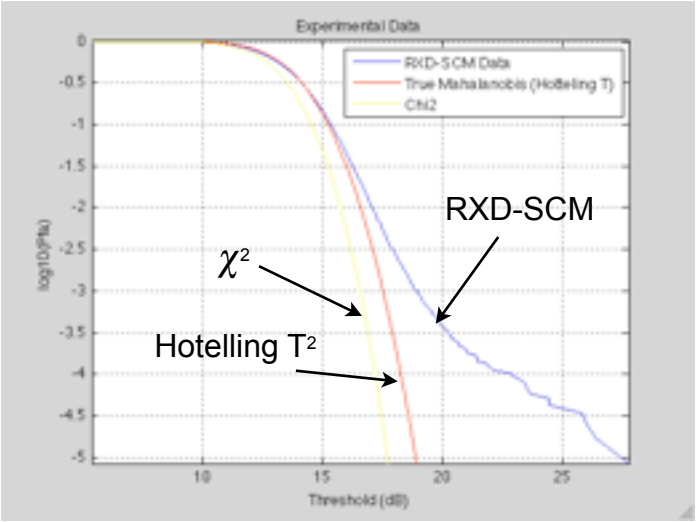
- Previous and new researches conducted in SONDRA-Lab in Non-Gaussian Detection and Estimation (J.P. Ovarlez and F. Pascal):
  - **Elliptical distributions (generalization of the SIRV distributions),**
  - **Random Matrix Theory (closely related to F. Pascal's talk).**

- **Mélanie Mahot** Ph.D. (2009-2012): Evaluation of the robust statistic framework for radar signal processing:
  - **M-estimators:** robust covariance matrix estimation in the context of Elliptical Distributions,
  - **Statistical properties** of M-estimators (useful for deriving tests PDF, ...).

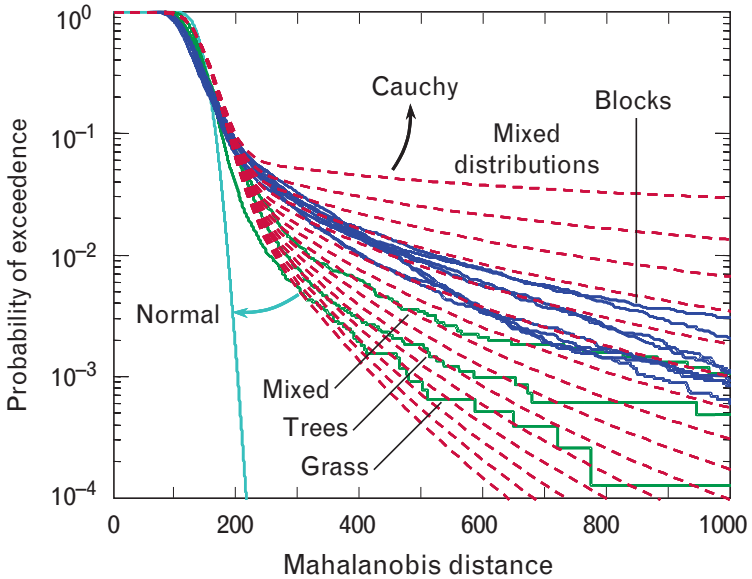
- **Pierre Formont** Ph.D. (2009-2012): Classification techniques applied to POLSAR, POLINSAR imaging:
  - **Statistical tests** based on covariance matrices for multivariate SAR classification, SAR change detection,
  - Use of the **Geometry of Information**,
  - 3 months attachment at DSO in 2012 (extension of polarimetric SAR classification algorithms to Hyperspectral Imaging).

FIRST COMMENTS AND ADEQUACY WITH SOME RESULTS FOUND IN THE LITERATURE

- Hyperspectral data provided by DSO (or others!) are **spatially heterogeneous** in intensity and/or **cannot be only characterized by Gaussian statistic**:



RXD on DSO DATA



[*Manolakis 2002*]

$$RXD_{SCM} = (\mathbf{c}_k - \boldsymbol{\mu})^H \, \hat{\mathbf{M}}_{SCM}^{-1} \, (\mathbf{c}_k - \boldsymbol{\mu})$$

- Spherical Invariant Random Vectors (SIRV) models have been started to be studied in the hyperspectral scientific community but one still uses .... Gaussian estimates !

SIRV FOR HYPERSPECTRAL BACKGROUND MODELLING

Spherically Invariant Random Vector : Compound Gaussian Process [*Yao 1973*]

$$\mathbf{c} = \sqrt{\tau}\,\mathbf{x} \qquad p_m(\mathbf{c}) = \int_0^{+\infty} \frac{1}{(\pi\,\tau)^m\,|\mathbf{M}|}\,\exp\left(-\frac{\mathbf{c}^H\,\mathbf{M}^{-1}\,\mathbf{c}}{\tau}\right)\,p(\tau)\,d\tau$$

- **x is a multivariate complex circular zero-mean Gaussian *m*-vector (speckle) with covariance matrix M with identifiability consideration tr(M)=m,**
- **τ is a positive scalar random variable (texture) well defined by its pdf p(τ).**

- **For a given set of spatial pixels of the hyperspectral image, M characterizes the correlation existing within the spectral bands,**
- **Conditionally to the pixel, the spectral vector is Gaussian. The texture variable τ characterizes here the variation of the norm of each vector from pixels to pixels.**

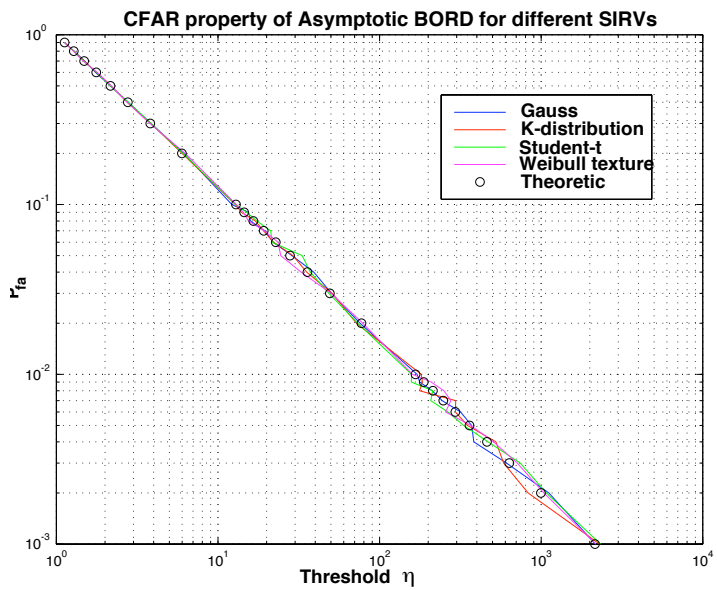
Powerful statistical model that allows:

- **to encompass the Gaussian model,**
- **to extend the Gaussian model (K, Weibull, Fisher, Cauchy, Alpha-Stable, ...),**
- **to take into account the heterogeneity of the background power with the texture,**
- **to take into account possible correlation existing on the m-channels of observation,**
- **to derive optimal or suitable detectors.**

SIRV DETECTORS

Detectors developed in the SIRV context

- **SIRV texture p(τ) modelling with Padé Approximants [*Jay et al., 2000*],**
- **Normalized Matched Filter [*Picinbono 1970, Scharf 1991*], GLRT-LQ [*Gini, Conte,1995*],**
- **Bayesian estimation (BORD) of the SIRV texture p(τ) [*Jay et al., 2002*].**



Texture-CFAR property for the GLRT-LQ

Normalized Matched Filter

$$\Lambda(\mathbf{c}) = \frac{\left|\mathbf{p}^H \mathbf{M}^{-1} \mathbf{c}\right|^2}{\left(\mathbf{p}^H \mathbf{M}^{-1} \mathbf{p}\right)\left(\mathbf{c}^H \mathbf{M}^{-1} \mathbf{c}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

c: cell under test
p: spectral steering vector of the target

Λ(c) is SIRV CFAR
but needs to know the true covariance M

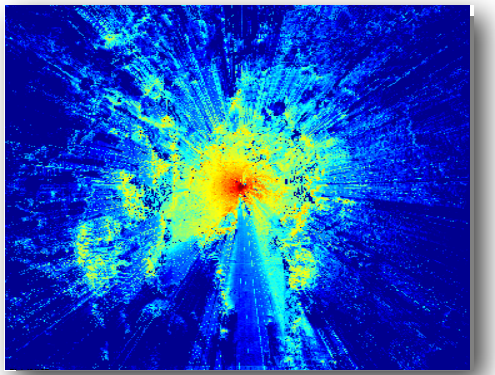
ADATIVE DETECTION IN SIRV BACKGROUND

New detectors called Adaptive Detectors can be derived by replacing in the NMF a «*good estimate*» of the covariance matrix (two step GLRT).

ACE : Adaptive Coherence Estimator
ANMF : Adaptive Normalized Matched Filter

$$\Lambda(\mathbf{y}) = \frac{\left|\mathbf{p}^H \hat{\mathbf{M}}^{-1} \mathbf{y}\right|^2}{\left(\mathbf{y}^H \hat{\mathbf{M}}^{-1} \mathbf{y}\right)\left(\mathbf{p}^H \hat{\mathbf{M}}^{-1} \mathbf{p}\right)} \underset{H_1}{\overset{H_0}{\lessgtr}} \lambda$$

These detectors are SIRV-CFAR only for some particular estimates of M !



$$\hat{\mathbf{M}} \; ???$$

Some well known estimates

$$\hat{\mathbf{M}}_{SCM} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{c}_k \, \mathbf{c}_k^H$$

$$\hat{\mathbf{M}}_{NSCM} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \mathbf{c}_k}$$

[Gini-Conte, 2002]

CHOICE OF THE COVARIANCE MATRIX ESTIMATE

- **The Sample Covariance Matrix SCM may be a «poor» estimate of the SIRV Covariance Matrix M because of the texture contamination:**

$$\hat{\mathbf{M}}_{SCM} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{c}_k\,\mathbf{c}_k^H = \frac{1}{K}\sum_{k=1}^{K} \tau_k\,\mathbf{x}_k\,\mathbf{x}_k^H \neq \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_k\,\mathbf{x}_k^H$$

- **The Normalized Sample Covariance Matrix (NSCM) may be a good candidate of the SIRV Covariance Matrix M:**

$$\hat{\mathbf{M}}_{NSCM} = \frac{m}{K}\sum_{k=1}^{K} \frac{\mathbf{c}_k\,\mathbf{c}_k^H}{\mathbf{c}_k^H\,\mathbf{c}_k} = \frac{m}{K}\sum_{k=1}^{K} \frac{\mathbf{x}_k\,\mathbf{x}_k^H}{\mathbf{x}_k^H\,\mathbf{x}_k}$$

This estimate does not depend on the texture but it is biased ($E\left[\hat{\mathbf{M}}_{NSCM}\right]$ and M share the same eigenvectors but have different eigenvalues, with the same ordering) [*Bausson et al. 2006*].

COVARIANCE MATRIX ESTIMATION IN SIRV/ ELLIPTICAL BACKGROUND

For an unknown but deterministic texture parameter, the Maximum Likelihood Estimate (MLE) of the Covariance M (approached MLE in the SIRV context), called the Fixed Point M_{FP} (FP), is the solution of the following implicit equation [*Conte-Gini 2002*]:

Fixed Point (FP):

$$\hat{\mathbf{M}}_{FP} = \frac{m}{K} \sum_{k=1}^{K} \frac{\mathbf{c}_k \, \mathbf{c}_k^H}{\mathbf{c}_k^H \, \hat{\mathbf{M}}_{FP}^{-1} \, \mathbf{c}_k}$$

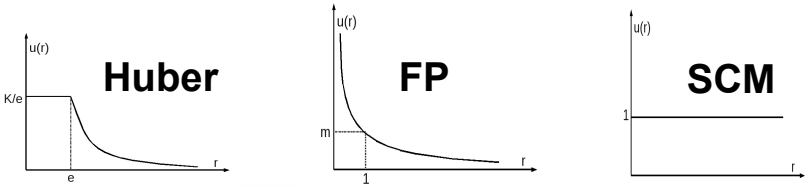
[*Pascal et al. 2006*]

- **This estimate does not depend on the texture,**
- **The Fixed Point is consistent, unbiased, asymptotically Gaussian and is, at a fixed number K, Wishart distributed with *mK/(m+1)* degrees of freedom,**
- **Existence and unicity of the solution are proven. The solution can be reached by recurrence $M_k=f(M_{k-1})$ whatever the starting point M_0 (ex: $M_0=I$, $M_1=M_{NSCM}$),**
- **Robust to outliers, strong targets or scatterers in the reference cells.**

The Fixed Point belongs to the family of M-estimators (*Robust Statistics [Huber 1964, Maronna 1976, Yohai 2006*]) in the more general context of Elliptically Random Process:

$$\hat{\mathbf{M}} = \frac{1}{K} \sum_{k=1}^{K} u\left(\mathbf{c}_k^H \, \hat{\mathbf{M}}^{-1} \, \mathbf{c}_k\right) \mathbf{c}_k \, \mathbf{c}_k^H$$

***u*(.) choice**

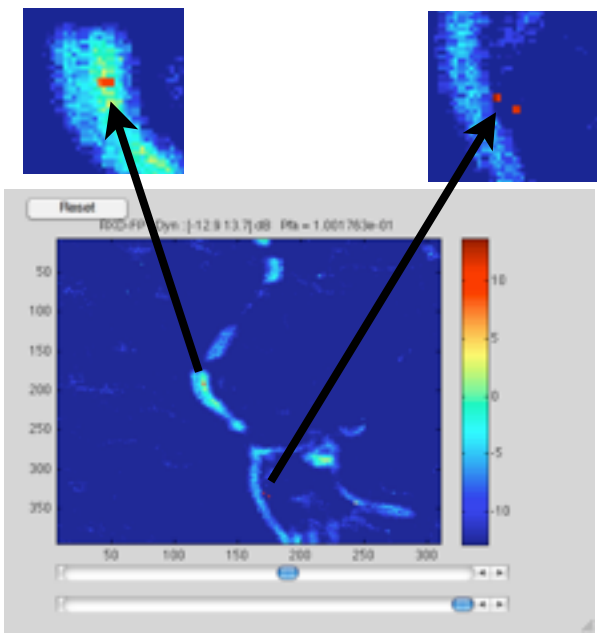


TEXTURE ESTIMATION IN SIRV BACKGROUND

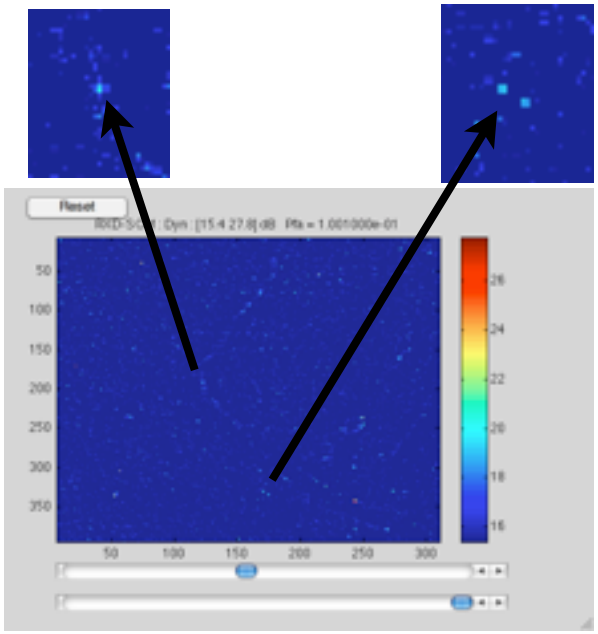
For an unknown but deterministic texture parameter, the Maximum Likelihood Estimate of the texture at pixel *k* is given by:

$$\hat{\tau}_k = \frac{\mathbf{c}_k^H \, \hat{\mathbf{M}}_{FP}^{-1} \, \mathbf{c}_k}{m}$$

This quantity plays exactly the same role as the **Polarimetric Whitening Filter** [*Novak and Burl 1990 - Vasile, J.P. Ovarlez et al. 2010*] for reducing the speckle in Polarimetric SAR images. It can also be seen as an **extended *Mahalanobis distance*** between $\mathbf{c}_k$ and the background.



$$RXD_{FP} = (\mathbf{c}_k - \boldsymbol{\mu})^H \, \hat{\mathbf{M}}_{FP}^{-1} \, (\mathbf{c}_k - \boldsymbol{\mu})$$



$$RXD_{SCM} = (\mathbf{c}_k - \boldsymbol{\mu})^H \, \hat{\mathbf{M}}_{SCM}^{-1} \, (\mathbf{c}_k - \boldsymbol{\mu})$$

SOME PROBLEMS SOLVED IN HYPERSPECTRAL CONTEXT

The hyperspectral data are real and positive as they represent radiance or reflectance.

- A mean vector has to be included in the SIRV model and to be estimated jointly with the covariance matrix,
- The real data can be transformed into complex ones by a linear Hilbert filter.

$$p_m(\mathbf{c}) = \int_0^{+\infty} \frac{1}{(\pi\,\tau)^m\,|\mathbf{M}|}\,\exp\left(-\frac{(\mathbf{c}-\boldsymbol{\mu})^H\,\mathbf{M}^{-1}\,(\mathbf{c}-\boldsymbol{\mu})}{\tau}\right)\,p(\tau)\,d\tau$$

Joint MLE solutions are [*Bilodeau 1999*]:

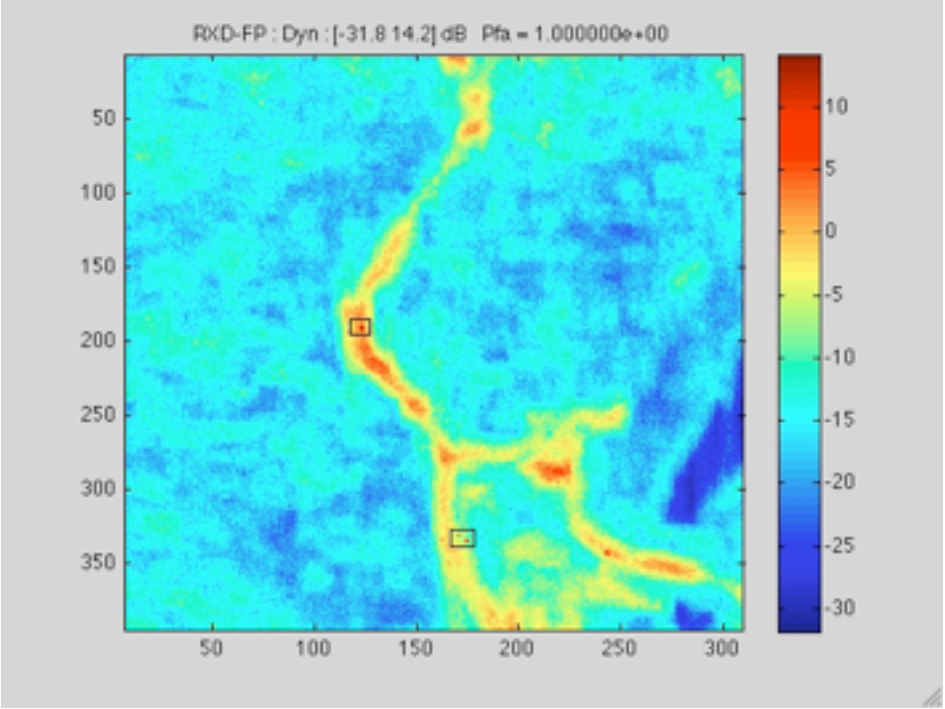
$$\hat{\mathbf{M}}_{FP} = \frac{m}{K}\sum_{k=1}^{K}\frac{(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})\,(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})^H}{(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})^H\,\hat{\mathbf{M}}_{FP}^{-1}\,(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})}$$

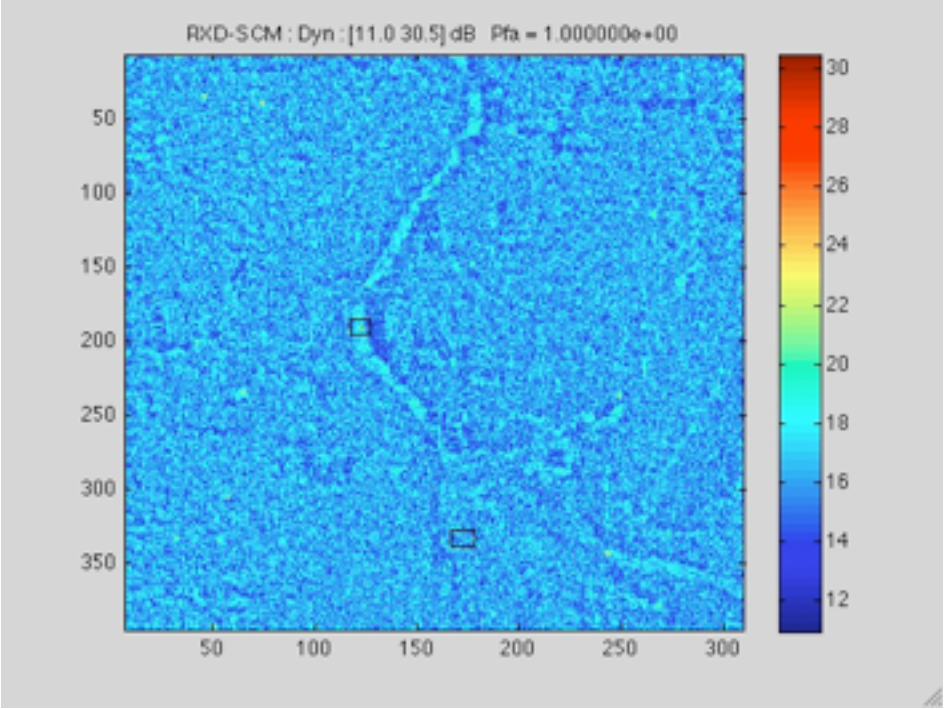
$$\widehat{\boldsymbol{\mu}} = \frac{\displaystyle\sum_{k=1}^{K}\frac{\mathbf{c}_k}{(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})^H\,\hat{\mathbf{M}}_{FP}^{-1}\,(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})}}{\displaystyle\sum_{k=1}^{K}\frac{1}{(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})^H\,\hat{\mathbf{M}}_{FP}^{-1}\,(\mathbf{c}_k-\widehat{\boldsymbol{\mu}})}}$$

These two quantities can be jointly reached by iterative process

FIRST RESULTS FOR ANOMALY DETECTION (DSO DATA)

Local Covariance Matrix estimate approach





$$RXD_{FP} = (\mathbf{c}_k - \widehat{\boldsymbol{\mu}})^H \, \hat{\mathbf{M}}_{FP}^{-1} \, (\mathbf{c}_k - \widehat{\boldsymbol{\mu}})$$

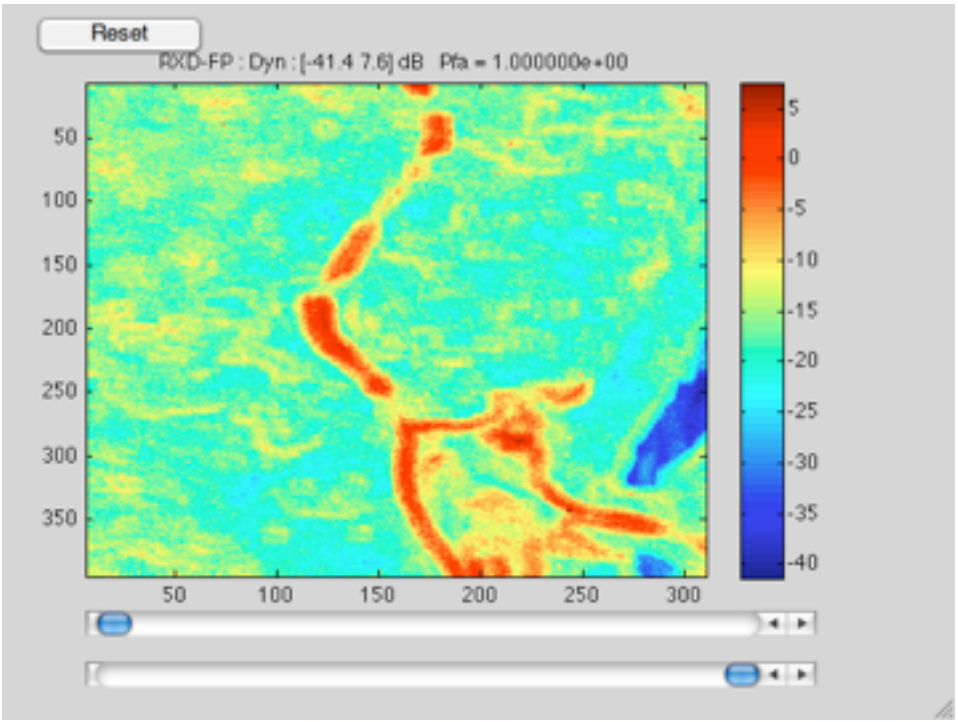
$$RXD_{SCM} = (\mathbf{c}_k - \widehat{\boldsymbol{\mu}})^H \, \hat{\mathbf{M}}_{SCM}^{-1} \, (\mathbf{c}_k - \widehat{\boldsymbol{\mu}})$$

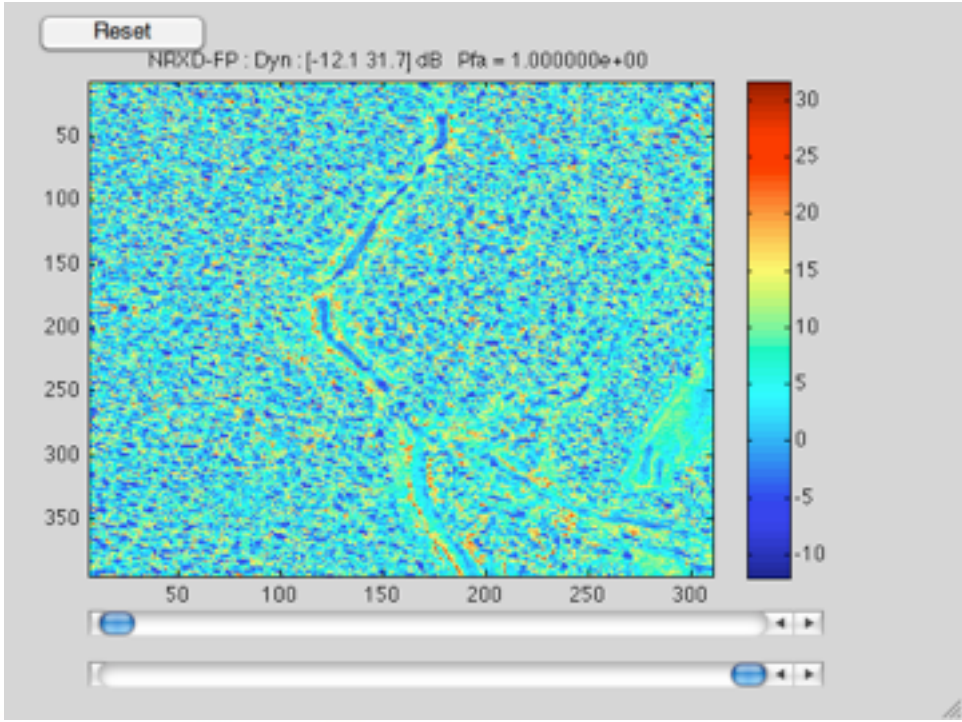
[*Reed and Yu, 1990*]

CONSIDERATIONS ON MAHALANOBIS DISTANCE

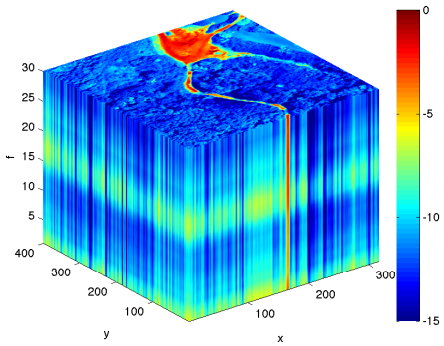
Mahalanobis Distance (RXD) built with SCM or FP still depends on the texture of the cell under test ! A solution may be to seek for a candidate which is invariant with the texture. Example, Mahalanobis distance built on the normalized cell under test:

$$NRXD_{FP} = \frac{(\mathbf{c}_k - \widehat{\boldsymbol{\mu}}_k)^H \, \hat{\mathbf{M}}_{FP}^{-1} \, (\mathbf{c}_k - \widehat{\boldsymbol{\mu}}_k)}{(\mathbf{c}_k - \widehat{\boldsymbol{\mu}}_k)^H \, (\mathbf{c}_k - \widehat{\boldsymbol{\mu}}_k)}$$



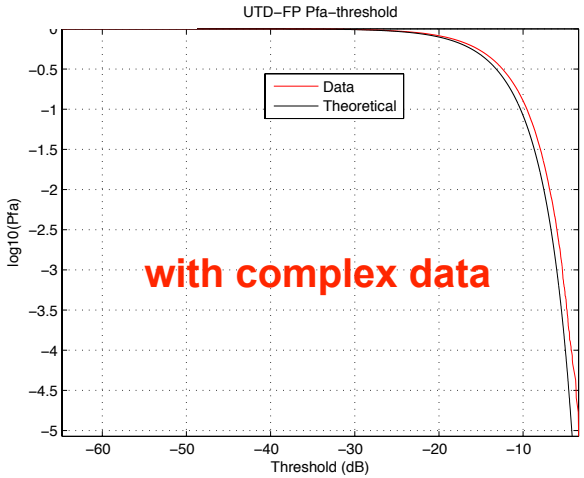


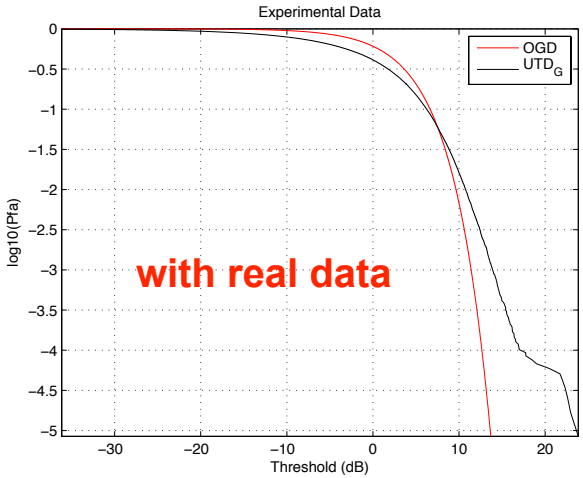
FALSE ALARM REGULATION FOR THE DETECTION



SIRV-CFAR TEST

$$\Lambda(\hat{\mathbf{M}}_{FP}, \widehat{\boldsymbol{\mu}}) = \frac{\left|\mathbf{p}^H \hat{\mathbf{M}}_{FP}^{-1} (\mathbf{c} - \widehat{\boldsymbol{\mu}})\right|^2}{\left(\mathbf{p}^H \hat{\mathbf{M}}_{FP}^{-1} \mathbf{p}\right)\left((\mathbf{c} - \widehat{\boldsymbol{\mu}})^H \hat{\mathbf{M}}_{FP}^{-1} (\mathbf{c} - \widehat{\boldsymbol{\mu}})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$





ACE - FP **AMF - SCM**

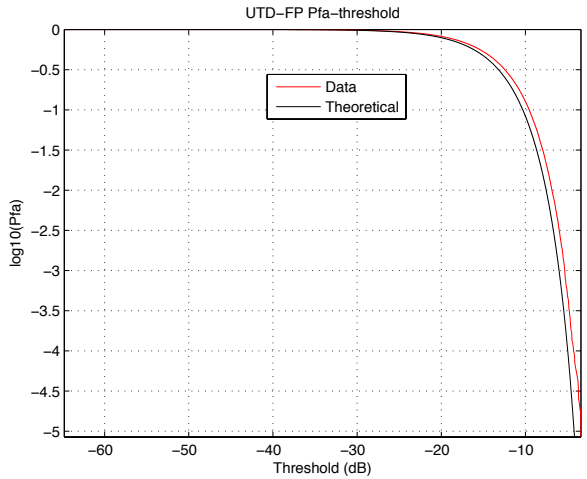
$$P_{fa} = (1-\lambda)\frac{m}{m+1}^{K-m+1} {}_2F_1\left(\frac{m}{m+1}K - m + 2, \frac{m}{m+1}K - m + 1; \frac{m}{m+1}K + 1; \lambda\right)$$

FALSE ALARM REGULATION FOR THE DETECTION

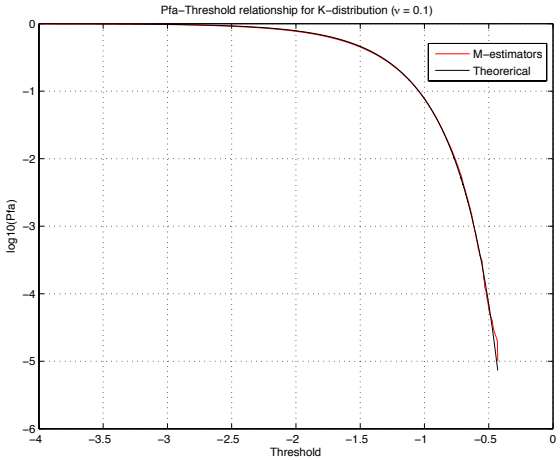
Robust Detection and Classification in Hyperspectral Imaging

PH.D. started in Oct. 2011, in collaboration with F. Pascal (SONDRA) and GIPSA-Lab (J. Chanussot)

Goal: to analyze and to extend previous works in the context of Elliptical distributions for robust scale (covariance matrix) and location (mean vector) M-estimation.



ACE - FP (2010 result)



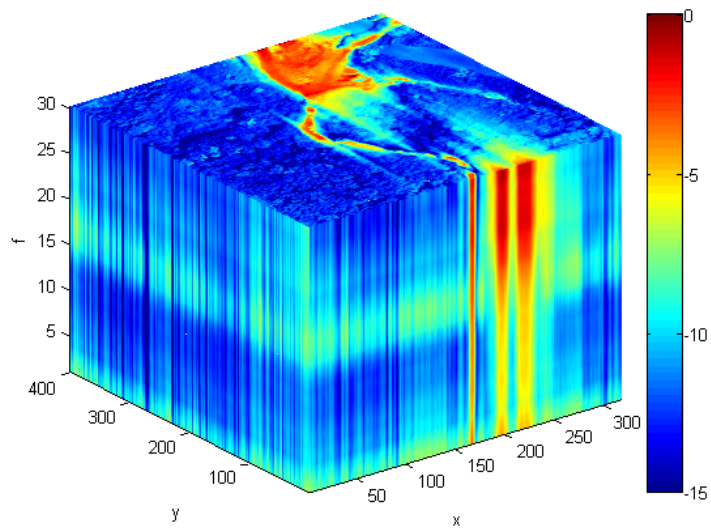
New ACE - M-estimate (2012 result)

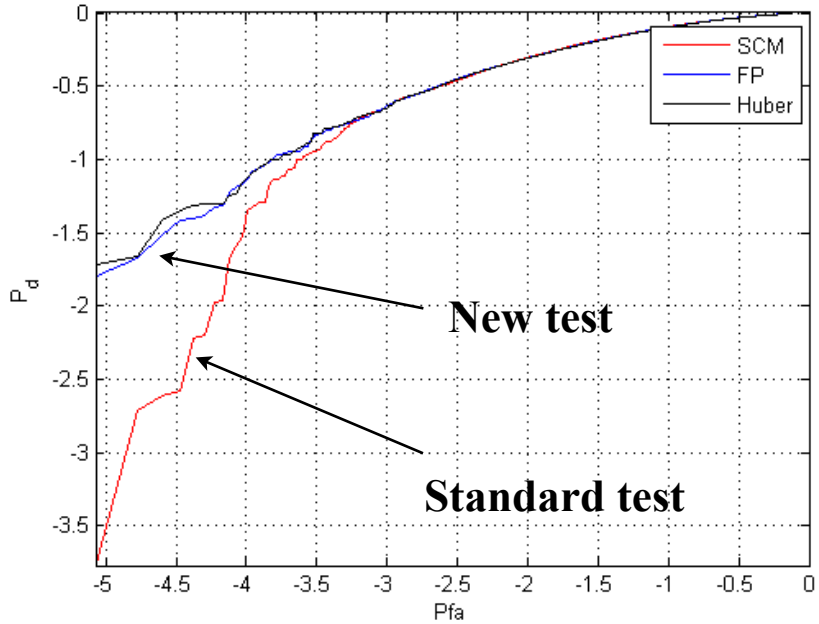
$$P_{fa} = (1-\lambda)^{\frac{K-1}{\sigma_1} - m + 1} \, {}_2F_1\left(\frac{K-1}{\sigma_1} - m + 2, \frac{K-1}{\sigma_1} - m + 1; \frac{K-1}{\sigma_1} + 1; \lambda\right)$$

The asymptotic statistical analysis of M-estimates allows to enhance the regulation of false alarm (*IEEE WHISPER 2012, IEEE IGARSS 2012*).

DETECTION PERFORMANCES ON EXPERIMENTAL DATA

Detection result on DSO experimental Hyperspectral Data





ACE test: $$\Lambda(\hat{\mathbf{M}}, \widehat{\boldsymbol{\mu}}) = \frac{|\mathbf{p}^{\mathbf{H}}\hat{\mathbf{M}}^{-1}(\mathbf{y} - \widehat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^{\mathbf{H}}\hat{\mathbf{M}}^{-1}\mathbf{p})((\mathbf{y} - \widehat{\boldsymbol{\mu}})^H\hat{\mathbf{M}}^{-1}(\mathbf{y} - \widehat{\boldsymbol{\mu}}))} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

with a well chosen joint $\hat{\mathbf{M}}$ and $\hat{\boldsymbol{\mu}}$ estimates (Huber, fixed point, ...)

CONCLUSIONS

- Hyperspectral images like radar data or SAR images can suffer from non-Gaussianity or heterogeneity that can reduce the performance of anomaly detectors (RXD), target detectors (ACE) and classifiers,

- SIRV/Elliptical modelling is a very nice theoretical tool for hyperspectral context that can match and control the heterogeneity and non-Gaussianity of the images,

- Jointly used with powerful and robust estimates, hyperspectral detectors may provide better performances, with nice CFAR properties,

- When extended to the Hyperspectral context, all the methodologies (Random Matrix Theory, non-Gaussian modelling, robust estimation, ...) previously and currently developed in SONDRA for radar applications can enhance performance in detection and classification problems:
    - source localization,
    - linear spectral unmixing, sub-spaces techniques,
    - detection, estimation, classification, ...