HyperImage Concept: Multidimensional Time-Frequency Analysis Applied to SAR Imaging

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Set of problems

Classical radar imaging makes the assumption that the scatterers are ponctual, isotropic and non-dispersive (model of "bright points"). However new very high resolution (VHR) SAR imaging cannot venture this hypothesis. Figure 1 highlights the dispersive behavior of some scatterers for a SAR image built with three equal subbands centered on the frequencies $f_c = 8.82$ GHz, $f_c = 9.37$ GHz and $f_c = 10$ GHz.

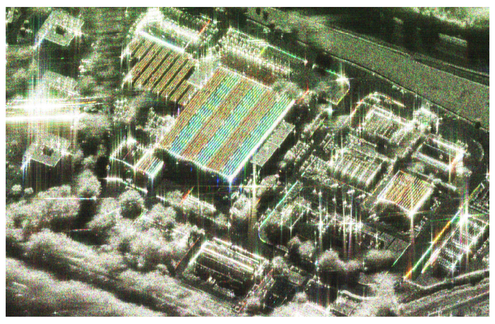

Fig.1: A SAR image which highlights dispersive scatterers.

The usually spatial repartition of scatterer $I(\vec{r})$ where $\vec{r}$ is the location of the scatterer, has now to depend on the wave vector $\vec{k}$. This new repartition is noted $I(\vec{r},\vec{k})$ and called HyperImage.

Affine Class

A family of functions is built $\Psi_{\mathbf{r}_0,\mathbf{k}_0}$ from the mother wavelet $\phi(\mathbf{k})$ by the similarity group $\mathcal{S}$:

$$\Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k}) = \frac{1}{k_o} e^{-j2\pi\mathbf{k}.\mathbf{r}_o}\phi\left(\frac{1}{k_o}\mathcal{R}_{\theta_o}^{-1}\mathbf{k}\right) = \frac{1}{k_o} e^{-j2\pi\mathbf{k}.\mathbf{r}_o}\,\phi\left(\frac{k}{k_o},\theta-\theta_o\right).$$

The wavelet coefficient $C_H(\mathbf{r}_o,\mathbf{k}_o)$ is defined as the scalar product between the complex backscattering coefficient H and the wavelet $\Psi_{\mathbf{r}_o,\mathbf{k}_o}$:

$$C_H(\mathbf{r}_o,\mathbf{k}_o) =< H, \Psi_{\mathbf{r}_o,\mathbf{k}_o} >$$

The scalar product is defined according to:

$$C_H(\mathbf{r}_o,\mathbf{k}_o) = \int_0^{2\pi} d\theta \int_0^{+\infty} \frac{k}{k_o} H(k,\theta)\, e^{j2\pi\mathbf{k}.\mathbf{r}_o}\phi^*\left(\frac{k}{k_o},\theta-\theta_o\right) dk$$

The scalogram which is the square modulus of the wavelet coefficients defines the hyperImage $\tilde{I}_H(\mathbf{r},\mathbf{k})$.

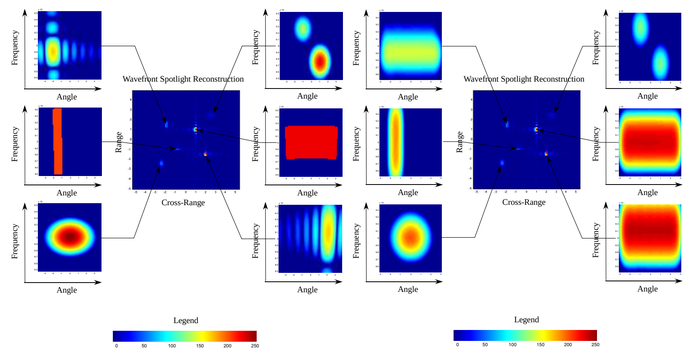

Fig. 2: Simulations results of the CWT

The continuous wavelet transform (CWT) has two interesting properties:

- Reconstruction

$$H(\mathbf{k}) = \frac{1}{K_\phi}\int_S d\mathbf{r}_o \int C_H(\mathbf{r}_o,\mathbf{k}_o)\,\Psi_{\mathbf{r}_o,\mathbf{k}_o}(\mathbf{k})\, d\mathbf{k}_o$$

where K_ϕ, called the *admissibility coefficient* has to verify:

$$K_\phi = \int |\phi(\mathbf{k})|^2 \frac{d\mathbf{k}}{k^2} + \infty$$

- Isometry:

$$\frac{1}{K_\phi}\int_S d\mathbf{r}_o \int |C_H(\mathbf{r}_o,\mathbf{k}_o)|^2\, d\mathbf{k}_o = \|H\|^2$$

Cohen Class

From the backscattering complex coefficient $H(\mathbf{k})$, by applying the classical bidimensional short time Fourier transform, the following hyperImage is defined:

$$F_H(\mathbf{r_0},\mathbf{k_0}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H(\mathbf{k})\, W^*(\mathbf{k}-\mathbf{k_0})\, e^{2i\pi\mathbf{k}\cdot\mathbf{r_0}}\, \mathrm{d}\mathbf{k}$$

where W is the Fourier transform of w. This transform allows to define the associated spectrogram:

$$S_H(\mathbf{r_0},\mathbf{k_0}) = |F_H(\mathbf{r_0},\mathbf{k_0})|^2$$

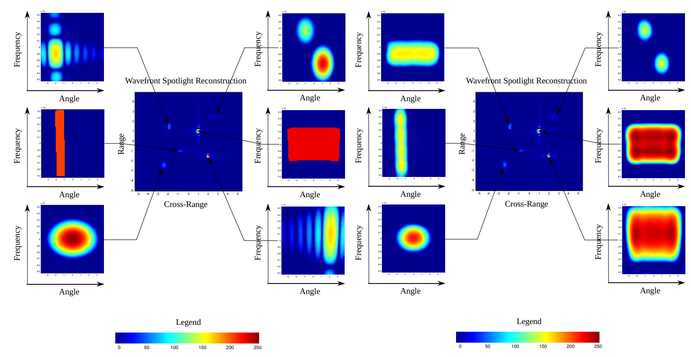

Fig. 3: Simulations results of the 2D Spectrogram

The spectrogram satisfies the energy conservation but not the marginal properties.

The resolution in spatial domain is limited by the window $w(\mathbf{r})$. Similarly, the resolution in the frequency and angle domains is limited by the width of the frequency window $W(\mathbf{k})$ (Heisenberg principle). The Gaussian window achieves the best resolution compromise among all the possible window function.

From the backscattering complex coefficient $H(\mathbf{k})$, by applying the classical bidimensional Wigner-Ville distribution, the following hyperImage is defined:

$$W_H(\mathbf{r_0},\mathbf{k_0}) = \int_{-\infty}^{+\infty} H\left(\mathbf{k_0}+\frac{\mathbf{k}}{2}\right) H^*\left(\mathbf{k_0}-\frac{\mathbf{k}}{2}\right) e^{2i\pi\mathbf{k}\cdot\mathbf{r_0}}\, \mathrm{d}\mathbf{k}$$

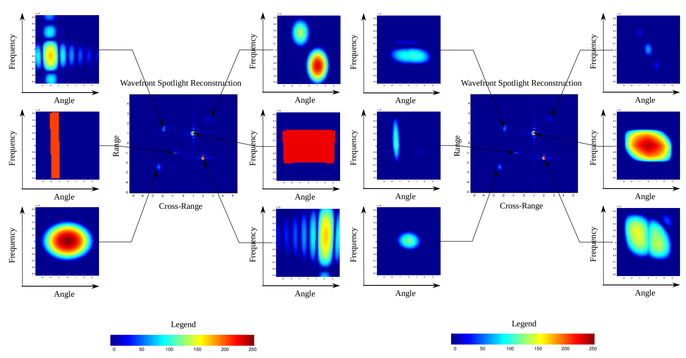

Fig.4: Simulations results of the 2D Wigner-Ville Transform

The Wigner-Ville distribution is real but non-positive in all the time-frequecy plane. It is a pseudo distribution of energy. The Wigner-Ville distribution has a number of desirable properties. It satisfies the marginal conditions (in spatial and frequency). It satisfies the energy conservation. It checks the instantaneous frequency property.

Although, the Wigner-Ville distribution has many nice properties, and gives nearly the best resolution among all the time-frequency techniques, its main drawback comes from cross-term interferences. Indeed, the Wigner-Ville transform of the sum of two signals is not the sum of the Wigner-Ville distributions.

Let the backscattering complex coefficient H received by the radar and it is the sum of coefficients H_1 et H_2 backscattered by two reflectors: $H(\mathbf{k}) = H_1(\mathbf{k}) + H_2(\mathbf{k})$. So, the Wigner-Ville distribution of this signal is explained by:

$$W_H(\mathbf{r},\mathbf{k}) = W_{H_1}(\mathbf{r},\mathbf{k}) + W_{H_2}(\mathbf{r},\mathbf{k}) + 2\mathcal{R}_e\left[\int_{\mathbb{R}^2} H_1\left(\mathbf{k}+\frac{\xi}{2}\right) H_2^*\left(\mathbf{k}-\frac{\xi}{2}\right) e^{j2\pi\xi\mathbf{r}}\, d\xi\right]$$

So, two scatterers create cross-term interferences. The solution is to filter the Wigner-Ville transform to suppress cross-term interferences.

Results

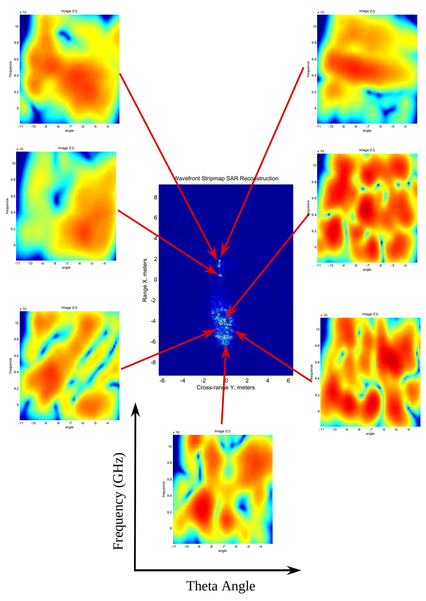

Fig.5: Time-frequency signatures extracted of the 2D Spectrogram

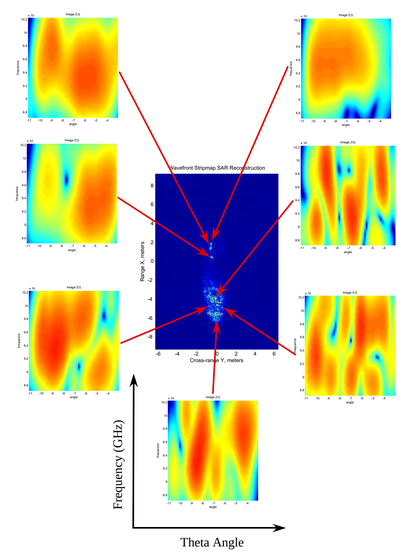

Fig.6: Time-frequency signatures extracted of the 2D continuous wavelet

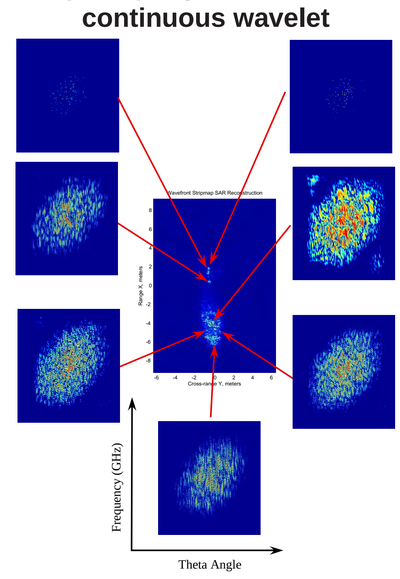

Fig.7: Time-frequency signatures extracted of the 2D Wigner-Ville Transform

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