Anomaly Detection and Estimation in Hyperspectral Imaging using Random Matrix Theory tools

Eugenie Terreaux [1] Frederic Pascal [2] Jean-Philippe Ovarlez [1]

[1] SONDRA/Suplec, 3 rue Joliot Curie, 91190 Gif-sur-Yvette, France
[2] LSS, CNRS, 3 rue Joliot Curie, 91190 Gif-sur-Yvette, France

Introduction and Motivations

Let us consider a hyperspectral image of Gaussian independent and identically distributed data, N **spatial** dimension and m **spectral** dimension

GOAL: Detecting and Estimating the number of anomalies on a Hyperspectral Image

- Large number of data: N and m are of same order with possibly $N > m$

$\Rightarrow$ If $(N,m) \to \infty$
Then **Law of Large Number** not valid anymore

Statistical Model :

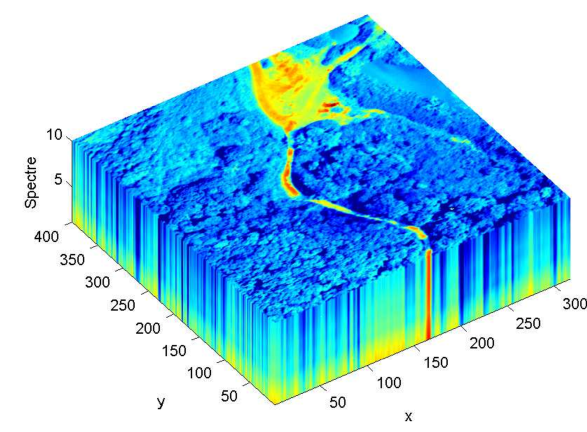
- Detect K independent anomalies among N observations

$\mathbf{x}_i$ independent gaussian noise vectors
$\mathbf{y}_i$ observation vectors
$*$ For the $K' = \sum K_i$ observations with anomalies:

$$\mathbf{y}_i = \sum_{j=1}^{K_i} \frac{\alpha_j}{\sqrt{m}} \mathbf{p}_j + \mathbf{x}_i \sim \mathcal{CN}\left(\frac{\alpha_j}{\sqrt{m}}\mathbf{p}, \mathbf{M}\right)$$

$*$ For the others:
$\mathbf{y}_i = \mathbf{x}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{M})$

Example

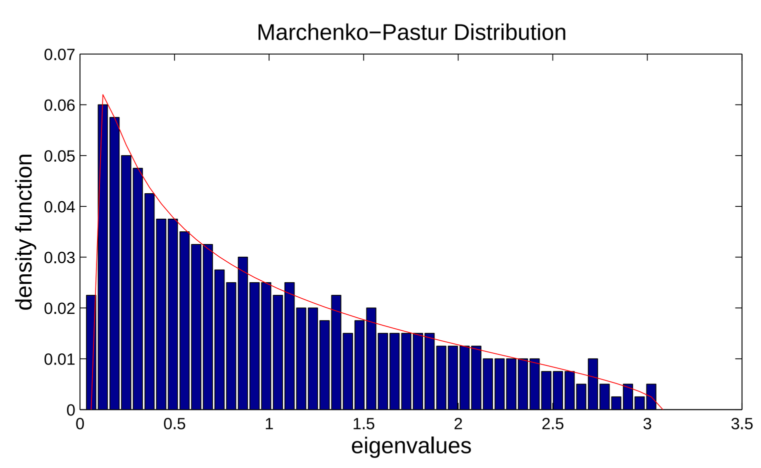


Hyperspectral image DSO National Laboratories.

Contribution : Using techniques of random matrix theory for hyperspectral images.

White Gaussian Noise

- **Sample Covariance Matrix** (SCM): $\hat{\mathbf{M}} = \frac{1}{N-1}\sum_{n=1}^{N} \mathbf{y}_i \mathbf{y}_i^H$
- SCM : Distribution of the eigenvalues $\to$ **Marchenko-Pastur** Law



Theorem : [1] $\boldsymbol{W}_N \in \mathbb{C}^{m\times N}$ **with independent identically distributed entries , mean=0 , variance=1**
If $N, m \to \infty$, $x \in$ **compact set**, $\lambda_{0,N}$ =**maxSpectre**$(\frac{1}{N}\boldsymbol{W}_N\boldsymbol{W}_N^H)$
Then

$$\mathbb{P}\left(m^{2/3}\frac{\lambda_{0,N} - b_N}{\sigma_N} \geq x\right) \to F_{TW}(x),$$

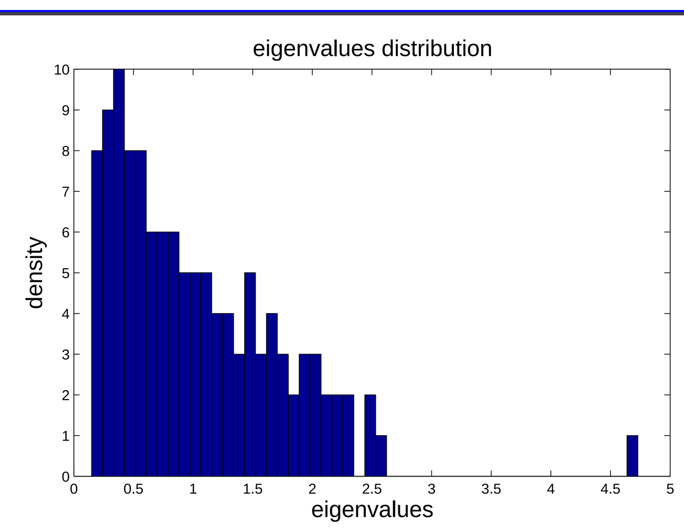
F_{TW} **Tracy-Widom distribution**
$c_N = m/N < 1$, $b_N = \left(1+\sqrt{c_N}\right)^2$, $\sigma_N = \left(1+\sqrt{c_N}\right) c_N^{4/3}$

- Hypothesis Test [2]

H_0 : at most k anomalies
H_1 : at least k anomalies

$$\hat{\lambda}_{k,N} \underset{H_0}{\overset{H_1}{\gtrless}} \left\{\hat{\sigma}^2(k)\left(b_N + \frac{\sigma_N}{m^{2/3}}\left(F_{TW}^{-1}(1-\alpha)\right)\right) = \zeta_N\right\}$$

$$\hat{K}_N = \mathrm{argmin}_k\left(\hat{\lambda}_{k,N} < \zeta_N\right) - 1.$$



Correlated Gaussian Noise

- Noise Sample Available (Anomaly free)

$*$ Whiten the signal
$*$ Same Test, b_N and σ_N different

- No Noise Sample Available

$*$ Find a gap between distance of two consecutive eigenvalues [2]

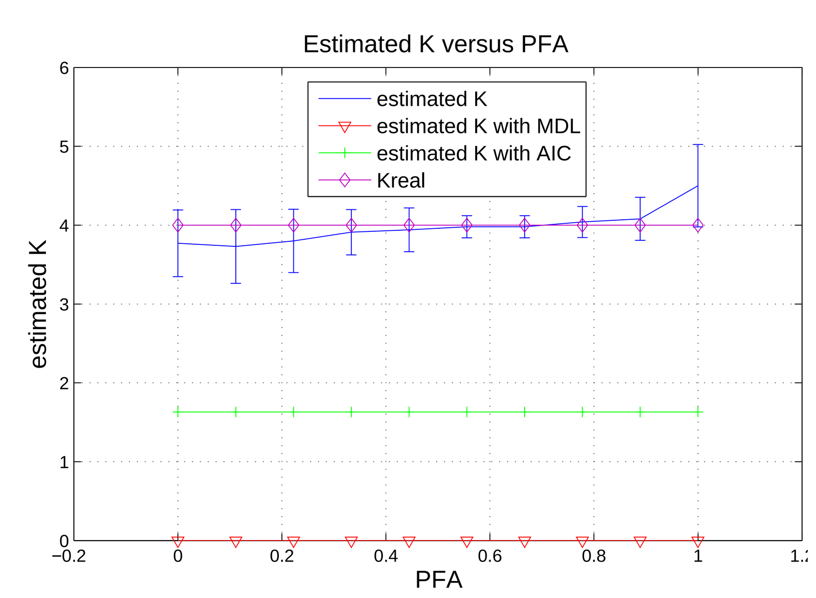
$$\hat{K}_N = \mathrm{argmax}_{k\in\{1,\ldots,L-1\}}\left(\frac{\hat{\lambda}_{k-1,N}}{\hat{\lambda}_{k,N}} > 1 + \text{threshold}\right)$$

with L $\geqslant$ K and $\lambda_{-1} = +\infty$

Experimental Results

- Gaussian noise anomaly free sample available

SNR	44	45	46	46.4	47
$\hat{K}_{estimean}$	0	1.2	3.1	4	4
Var	0	0.16	0.16	0	0
$\hat{K}_{AIC}$	0	0	2	2.9	4



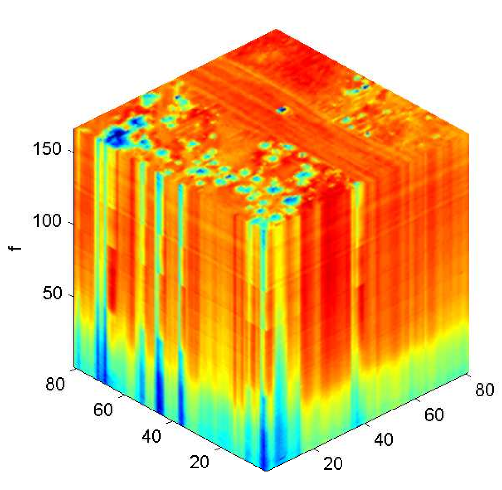
- No non-gaussian noise sample available

$*$ PFA-threshold relationship obtained with simulated data
$\Rightarrow$ Choice of the threshold

- Real image with a car to detect

	First eigenvalues ratios					
with car	37	3.4	4.3	3.6	1.1	2.0
without car	16	3.1	4.3	3.3	1.3	2.0

Threshold = 34.7 $\Rightarrow$ Car detected .



Conclusion

- Classical methods for anomaly detection $\to$ not adapted for large m and N
- Monte-Carlo simulations $\to$ illustrates improvement of this methods compared to AIC and MDL.
- Further works will address the problem of correlated and non-Gaussian noise.

[1] R. Couillet and M.bDebbah Random Matrix Methods for Wireless Communication, Cambridge University Press, 2011
[2] J. Vinogradova Random matrices and applications to detection and estimation in array processing, PhD thesis, Telecom ParisTech Paris, 2014

CentraleSupélec L2S SONDRA