Robust Estimation and Detection Schemes in Non-Standard Conditions for Radar, Array Processing and Imaging

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12ème École d'Été de Peyresq
en Traitement du Signal et des Images
25 juin au 01 juillet 2017

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Part B

Robust Detection and Estimation Schemes

Part B: Contents

Outline

Modeling the background

Let $\mathbf{z}$ be a complex circular random vector of length m. $\mathbf{z}$ has a Complex Elliptically Symmetric (CES) distribution ($CE(\boldsymbol{\mu}, \Sigma, g_.)$) if its PDF is [Kelker, 1970, Frahm, 2004, Ollila et al., 2012]:

$$g_{\mathbf{z}}(\mathbf{z}) = \pi^{-m} |\Sigma|^{-1} h_z((\mathbf{z}-\boldsymbol{\mu})^H \Sigma^{-1} (\mathbf{z}-\boldsymbol{\mu})), \tag{1}$$

where $h_z : [0,\infty) \to [0,\infty)$ is the density generator, where $\boldsymbol{\mu}$ is the statistical mean (generally known or $= \mathbf{0}$) and Σ is the scatter matrix. In general, $E\left[\mathbf{z}\,\mathbf{z}^H\right] = \alpha\,\Sigma$ where α is known.

- **Large class of distributions**: Gaussian ($h_z(z) = \exp(-z)$, SIRV, MGGD ($h_z(z) = \exp(-z^\alpha)$), etc.
- **Closed under affine transformations** (e.g. matched filter),
- **Stochastic representation theorem**: $\boxed{\mathbf{z} =_d \boldsymbol{\mu} + \mathcal{R}\mathbf{A}\mathbf{u}^{(k)}}$,

  where $\mathcal{R} \geq 0$, independent of $\mathbf{u}^{(k)}$ and $\Sigma = \mathbf{A}\mathbf{A}^H$ is a factorization of Σ, where $\mathbf{A} \in \mathbb{C}^{m\times k}$ with $k = \text{rank}(\Sigma)$.

SIRV: a CES subclass

The m-vector $\mathbf{z}$ is a complex Spherically Invariant Random Vector [Yao, 1973, Jay, 2002] if its PDF can be put in the following form:

$$g_{\mathbf{z}}(\mathbf{z}) = \frac{1}{\pi^m\,|\Sigma|} \int_0^{\infty} \frac{1}{\tau^m} \exp\left(\frac{(\mathbf{z}-\boldsymbol{\mu})^H\,\Sigma^{-1}\,(\mathbf{z}-\boldsymbol{\mu})}{\tau}\right) p_\tau(\tau)\,d\tau\,, \tag{2}$$

where $p_\tau : [0,\infty) \to [0,\infty)$ is the texture generator.

- **Large class of distributions**: Gaussian ($p_\tau(\tau) = \delta(\tau - 1)$), K-distribution ($p_\tau$ gamma), Weibull (no closed form), Student-t (p_τ inverse gamma), etc. Main Gaussian Kernel: closed under affine transformations,
- The texture random scalar is modeling the variation of the power of the Gaussian vector $\mathbf{x}$ along his support (e.g. heterogeneity of the noise along range bins, time, spatial domain, etc.),
- **Stochastic representation theorem**: $\boxed{\mathbf{z} =_d \boldsymbol{\mu} + \sqrt{\tau}\,\mathbf{A}\,\mathbf{x}}$, where $\tau \geq 0$ is the texture, independent of $\mathbf{x}$ and $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \Sigma)$.

Outline

Estimating the covariance matrix: Conventional estimators

Assuming n available SIRV secondary data $\mathbf{z}_k = \sqrt{\tau_k}\,\mathbf{x_k}$ where $\mathbf{x}_k \sim \mathcal{CN}(\mathbf{0}, \Sigma)$ and where τ_k scalar random variable.

- The **Sample Covariance Matrix** (SCM) may be a poor estimate of the Elliptical/SIRV Scatter/Covariance Matrix because of the texture contamination:

$$\hat{\mathbf{S}}_n = \frac{1}{n}\sum_{k=1}^{n} \mathbf{z}_k\,\mathbf{z}_k^H = \frac{1}{n}\sum_{k=1}^{n} \tau_k\,\mathbf{x}_k\,\mathbf{x}_k^H \neq \frac{1}{n}\sum_{k=1}^{n} \mathbf{x}_k\,\mathbf{x}_k^H\,,$$

- The **Normalized Sample Covariance Matrix** (NSCM) may be a good candidate of the Elliptical SIRV Scatter/Covariance Matrix:

$$\hat{\Sigma}_{NSCM} = \frac{1}{n}\sum_{k=1}^{n} \frac{\mathbf{z}_k\,\mathbf{z}_k^H}{\mathbf{z}_k^H\,\mathbf{z}_k} = \frac{1}{n}\sum_{k=1}^{n} \frac{\mathbf{x}_k\,\mathbf{x}_k^H}{\mathbf{x}_k^H\,\mathbf{x}_k},$$

  This estimate does not depend on the texture τ_k but it is biased and share the same eigenvectors but have different eigenvalues, with the same ordering [Bausson et al., 2007].

Estimating the covariance matrix

Let $(\mathbf{z}_1, ..., \mathbf{z}_n)$ be a n-sample $\sim CE_m(\mathbf{0}, \Sigma, g_{\mathbf{z}(.)})$ (Secondary data).

PDF $g_{\mathbf{z}}(.)$ specified: ML-estimator of Σ

$$\widehat{\Sigma} = \frac{1}{n} \sum_{i=1}^{n} \frac{-g_{\mathbf{z}}'\left(\mathbf{z}_i^H \widehat{\Sigma}^{-1} \mathbf{z}_i\right)}{g_{\mathbf{z}}\left(\mathbf{z}_i^H \widehat{\Sigma}^{-1} \mathbf{z}_i\right)} \mathbf{z}_i \mathbf{z}_i^H ,$$

PDF $g_{\mathbf{z}}(.)$ not specified: M-estimator of Σ

$$\widehat{\Sigma} = \frac{1}{n} \sum_{i=1}^{n} u\left(\mathbf{z}_i^H \widehat{\Sigma}^{-1} \mathbf{z}_i\right) \mathbf{z}_i \mathbf{z}_i^H ,$$

[Maronna et al., 2006, Kent and Tyler, 1991, Pascal, 2006, Pascal et al., 2008a, Pascal et al., 2008b]

- Existence, Uniqueness,
- Convergence of the recursive algorithm, etc.

Examples of M-estimators

SCM:

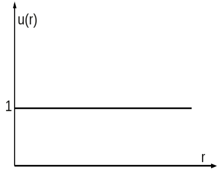
$$u(r) = 1$$

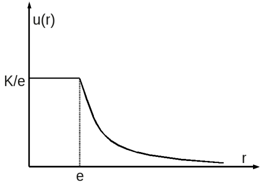
Huber's M-estimator:

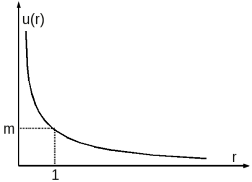
$$u(r) = \begin{cases} K/e \text{ if } r <= e \\ K/r \text{ if } r > e \end{cases}$$

FPE (Tyler):

$$u(r) = \frac{m}{r}$$







- Huber $=$ mix between SCM and FPE [Huber, 1964],
- FPE and SCM are "not" (theoretically) M-estimators,
- FPE is the most robust while SCM is the most efficient.

Estimating the covariance matrix: Tyler's M-estimators

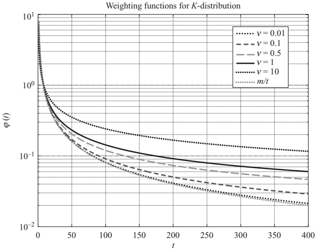
Let $(\mathbf{z}_1, ..., \mathbf{z}_n)$ be a n-sample $\sim CE_m(\mathbf{0}, \Sigma, g_{\mathbf{z}})$ (Secondary data).

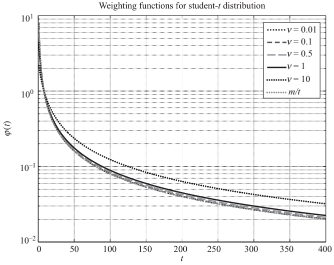
FP Estimate ([Tyler, 1987, Pascal et al., 2008a])

$$\widehat{\Sigma}_{FPE} = \frac{m}{n} \sum_{k=1}^{n} \frac{\mathbf{z}_k\, \mathbf{z}_k^H}{\mathbf{z}_k^H\, \widehat{\Sigma}_{FPE}^{-1}\, \mathbf{z}_k}.$$

- The FPE does not depend on the texture (SIRV or CES distributions),
- Existence, Uniqueness,
- Convergence of the recursive algorithm (identifiability condition: $\mathrm{tr}(\widehat{\Sigma}_{FPE}) = m$),
- True MLE under SIRV distributed noise with unknown deterministic texture $\{\tau_k\}_{k\in[1,n]}$.

Some Weighting Functions of M-estimators





$$\mathbf{u}(t) = \frac{\sqrt{\nu}}{t} \frac{K_{\nu-m-1}\left(4\,\nu\,t\right)}{K_{\nu-m}\left(4\,\nu\,t\right)}, \qquad \mathbf{u}(t) = \frac{\nu + 2\,m}{\nu + 2\,t}.$$

We have $\lim_{\nu \to 0} \hat{\Sigma} = \hat{\Sigma}_{FPE}$ and $\lim_{\nu \to \infty} \hat{\Sigma} = \hat{\Sigma}_{SCM}$.

Asymptotic distribution of complex M-estimators

Using the results of Tyler, we derived the following results [Mahot, 2012, Mahot et al., 2013]:

Theorem 1 (Asymptotic distribution of $\hat{\Sigma}$)

$$\sqrt{n}\,\mathrm{vec}(\hat{\Sigma} - \Sigma) \xrightarrow{d} \mathcal{CN}_{m^2}\left(\mathbf{0}_{m^2}, \mathbf{C}, \mathbf{P}\right), \quad (3)$$

where $\mathcal{CN}$ is the complex Gaussian distribution, $\mathbf{C}$ the CM and $\mathbf{P}$ the pseudo CM:

$$\begin{aligned} \mathbf{C} &= \sigma_1 \left(\Sigma^* \otimes \Sigma\right) + \sigma_2\,\mathrm{vec}(\Sigma)\mathrm{vec}(\Sigma)^H, \\ \mathbf{P} &= \sigma_1 \left(\Sigma^* \otimes \Sigma\right)\mathbf{K}_{m^2,m^2} + \sigma_2\,\mathrm{vec}(\Sigma)\mathrm{vec}(\Sigma)^T, \end{aligned}$$

where $\mathbf{K}_{m,m}$ is the $m \times m$ commutation matrix transforming any m-vector $\mathrm{vec}(\mathbf{A})$ into $\mathrm{vec}(\mathbf{A}^T)$ and where the constant σ_1 and σ_1 are completely defined.

An important property of complex M-estimators

- Let $\widehat{\Sigma}$ an estimate of Hermitian positive-definite matrix Σ that satisfies

$$\sqrt{n}\left(\text{vec}(\widehat{\Sigma}-\Sigma)\right) \xrightarrow{d} \mathcal{CN}\left(\mathbf{0}_m, \mathbf{C}, \mathbf{P}\right), \tag{4}$$

with

$$\begin{cases} \mathbf{C} = \nu_1\, \Sigma^* \otimes \Sigma + \nu_2\, \text{vec}(\Sigma)\,\text{vec}(\Sigma)^H, \\ \mathbf{P} = \nu_1\, (\Sigma^* \otimes \Sigma)\,\mathbf{K}_{m^2,m^2} + \nu_2\, \text{vec}(\Sigma)\,\text{vec}(\Sigma)^T, \end{cases}$$

where ν_1 and ν_2 are any real numbers.

e.g.

	SCM	M-estimators	FP
ν_1	1	σ_1	$(m+1)/m$
ν_2	0	σ_2	$-(m+1)/m^2$
...	More accurate		More robust

- Let $H(.)$ be a r-multivariate function on the set of Hermitian positive-definite matrices, with continuous first partial derivatives and such as $H(\mathbf{V}) = H(\alpha\mathbf{V})$ for all $\alpha > 0$, e.g. the ANMF statistic, the MUSIC statistic, etc.

Theorem 2 (Asymptotic distribution of $H(\widehat{\Sigma})$)

$$\sqrt{n}\left(H(\widehat{\Sigma}) - H(\Sigma)\right) \xrightarrow{d} \mathcal{CN}\left(\mathbf{0}_r, \mathbf{C}_H, \mathbf{P}_H\right), \tag{5}$$

where $\mathbf{C}_H$ and $\mathbf{P}_H$ are defined as

$$\begin{aligned}\mathbf{C}_H &= \nu_1\, H'(\Sigma)\left(\Sigma^T \otimes \Sigma\right) H'(\Sigma)^H,\\ \mathbf{P}_H &= \nu_1\, H'(\Sigma)\left(\Sigma^T \otimes \Sigma\right) \mathbf{K}_{m^2,m^2}\, H'(\Sigma)^T,\end{aligned}$$

where $H'(\Sigma) = \left(\dfrac{\partial H(\Sigma)}{\partial \mathrm{vec}(\Sigma)}\right)$.

CES distribution $\Rightarrow$ two-step GLRT ANMF

ANMF test (ACE, GLRT-LQ) [E. Conte and M. Lops and G. Ricci, 1995, Kraut and Scharf, 1999]

$$H(\widehat{\Sigma}) = \Lambda_{ANMF}(\mathbf{z}, \widehat{\Sigma}) = \frac{|\mathbf{p}^H \widehat{\Sigma}^{-1} \mathbf{z}|^2}{(\mathbf{p}^H \widehat{\Sigma}^{-1} \mathbf{p}) \, (\mathbf{z}^H \widehat{\Sigma}^{-1} \mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{ANMF} \,, \tag{6}$$

where $\widehat{\Sigma}$ stands for any M-estimators.

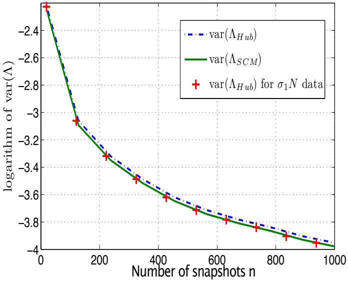
- The ANMF is scale-invariant (homogeneous of degree 0), i.e. $\forall \alpha, \beta \in \mathbb{R}, \, \Lambda_{ANMF}(\alpha \, \mathbf{z}, \beta \, \widehat{\Sigma}) = \Lambda_{ANMF}(\mathbf{z}, \widehat{\Sigma})$.
- Its asymptotic distribution (conditionally to $\mathbf{z}$!) is known [Pascal and Ovarlez, 2015, Ovarlez et al., 2015]

$$\sqrt{n} \, \left(H(\widehat{\Sigma}) - H(\Sigma) \right) \xrightarrow{d} \mathcal{CN} \left(0, \sigma_1 \, H(\Sigma) \, \left(H(\Sigma) - 1 \right)^2 \right) .$$

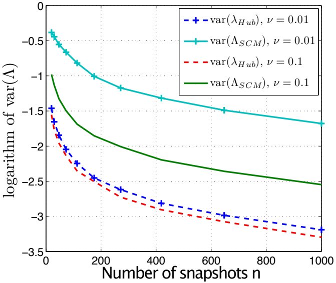
- It is CFAR w.r.t the covariance/scatter matrix,
- It is CFAR w.r.t the texture.

Illustrations of the result

- Complex Huber's M-estimator.
- Figure 1: Gaussian context, here $\sigma_1 = 1.066$.
- Figure 2: K-distributed clutter (shape parameter: $\nu = 0.1$ and 0.01).



Validation of theorem (even for small n)



Interest of the M-estimators

Some comments:

Perfect (but asymptotic) characterization of several objects properties, such as detectors, classifiers, estimators, etc.

$H(SCM)$ and $H(M\text{-estimators})$ share the same asymptotic distribution (differs from σ_1).

$$\Downarrow$$

- Link to the classical Gaussian case,
- Quantification of the loss involved by robust estimator.

Probability of false alarm

PFA-threshold relation of $\Lambda_{ANMF}(\widehat{\mathbf{S}}_n)$ (Gaussian case, finite n)

$$P_{fa} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,, \tag{7}$$

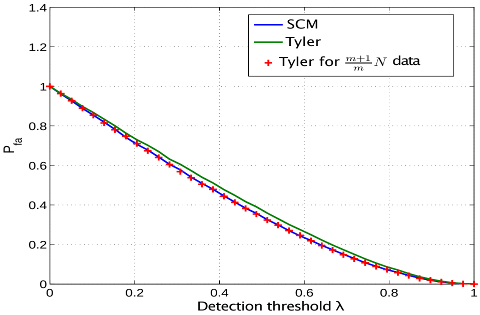
where $a = n - m + 2$, $b = n + 2$ and ${}_2F_1$ is the Hypergeometric function defined as

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)} \frac{x^k}{k!}\,.$$

Tyler's estimator: Gaussian context, $n = 10$, $m = 3$

PFA-threshold relation of Λ_{ANMF}(Tyler's est.) for CES distributions

For n large and any elliptically distributed noise, the PFA is still given by (7) if we replace n by $n/\frac{m+1}{m}$.



Probability of false alarm

For n large enough and for any elliptically distributed noise, the PFA is still given by (7) if we replace n by n/σ_1 [Pascal et al., 2004]:

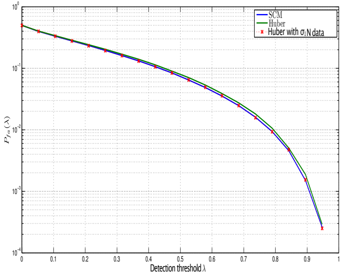
PFA-threshold relation of $\Lambda_{ANMF}(M\text{-est.})$ for CES distributions

$$P_{fa} = (1-\lambda)^{a-1}\, {}_2F_1(a, a-1; b-1; \lambda)\,, \tag{8}$$

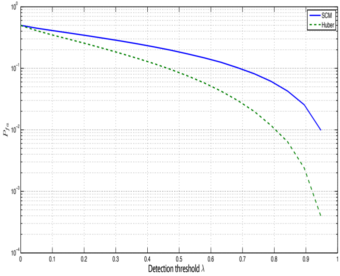
where $a = \dfrac{n}{\sigma_1} - m + 2$, $b = \dfrac{n}{\sigma_1} + 2$ and ${}_2F_1$ is the Hypergeometric function.

Illustrations of the result: Probabilities of False Alarm

- Complex Huber's M-estimator.
- Figure 1: Gaussian context, here $\sigma_1 = 1.066$.
- Figure 2: K-distributed clutter (shape parameter: $\nu = 0.1$).



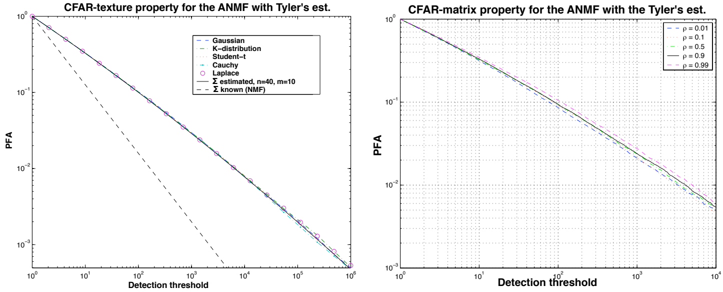
Validation of theorem (even for small n)



Interest of the M-estimators for False Alarm regulation

Illustration of the ANMF CFAR properties for CES process

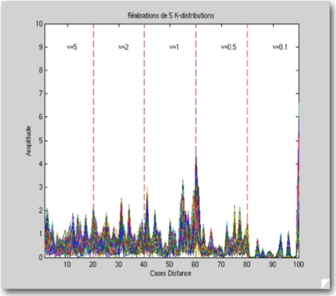
False Alarm regulation for ANMF built with Tyler's estimate

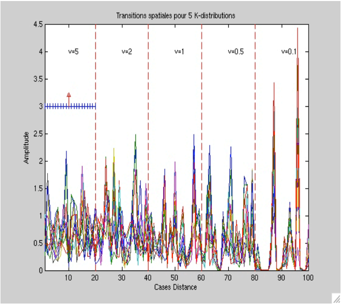


(a) CFAR-texture (b) CFAR-matrix

Figure: Illustration of the CFAR properties of the ANMF built with the Tyler's estimator, for a Toeplitz CM whose (i,j)-entries are $\rho^{|i-j|}$.

Properties of ANMF-Tyler Detector on Clutter Transitions

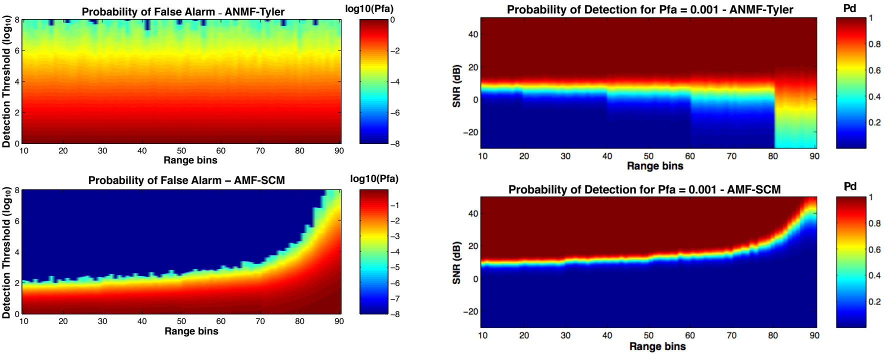




Play/Pause

- K-distributed clutter transitions: from Gaussian to impulsive noise,
- Estimation of the covariance matrix onto a range bins sliding window.

Properties of ANMF-Tyler Detector on Clutter Transitions



- ANMF-Tyler: The same detection threshold is guaranteed for a chosen P_{fa} whatever the clutter area,
- ANMF-Tyler: Performance in terms of detection is kept for moderate non-Gaussian clutter and improved for spiky clutter.

Outline

Robustness of the M-estimators

Let us suppose that $\{\mathbf{y}_i\}_{i=1,n-1} \sim \mathcal{CN}(\mathbf{0}, \Sigma)$ and that the last secondary data $\mathbf{y}_n$ contains outlier $\mathbf{p}_0$:

- Sample Covariance Matrix case:

$$\hat{\mathbf{S}}_n^{pol} = \frac{1}{n} \sum_{k=1}^{n-1} \mathbf{y}_k \, \mathbf{y}_k^H + \frac{1}{n} \mathbf{p}_0 \, \mathbf{p}_0^H \,, \qquad E\left[\hat{\mathbf{S}}_n^{pol}\right] = \frac{n-1}{n} \Sigma + \frac{1}{n} E\left[\mathbf{p}_0 \, \mathbf{p}_0^H\right] .$$

The power of the outlier $\mathbf{p}_0$ has a **big impact** on the quality of the SCM estimation.

- Tyler (or FP) Covariance Matrix case:

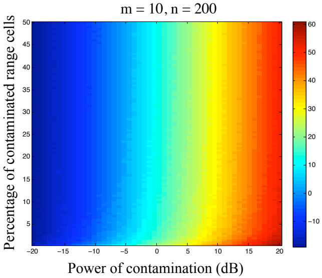
$$\hat{\Sigma}_{FPpol} = \frac{m}{n} \sum_{k=1}^{n} \frac{\mathbf{y}_k \, \mathbf{y}_k^H}{\mathbf{y}_k^H \, \hat{\Sigma}_{FPpol}^{-1} \, \mathbf{y}_k} \,, \qquad E\left[\hat{\Sigma}_{FPpol}\right] = \Sigma + \frac{m+1}{n} \left[E\left[\frac{\mathbf{p}_0 \, \mathbf{p}_0^H}{\mathbf{p}_0^H \, \Sigma^{-1} \, \mathbf{p}_0} \right] - \frac{1}{m} \Sigma \right] .$$

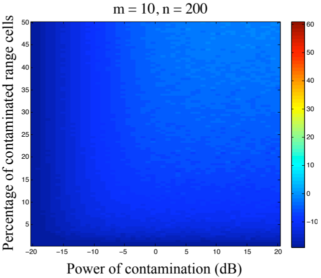
The power of the outlier $\mathbf{p}_0$ has **no big impact** on the quality of the Tyler estimate.

Robustness of M-estimators

Gaussian vectors $\mathbf{y}_k$ polluted by outliers

$$\hat{\mathbf{S}}_n = \frac{1}{n}\sum_{k=1}^{n} \mathbf{y}_i\,\mathbf{y}_k^H, \qquad \hat{\Sigma}_{FP} = \frac{m}{n}\sum_{k=1}^{n} \frac{\mathbf{y}_k\,\mathbf{y}_k^H}{\mathbf{y}_k^H\,\hat{\Sigma}_{FP}^{-1}\,\mathbf{y}_k}.$$

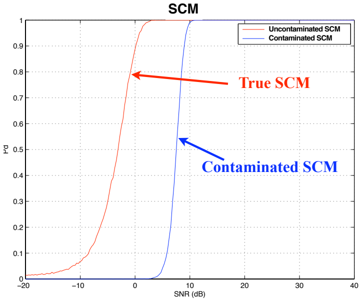




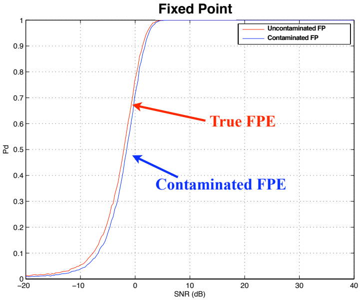
Plot of the error between the covariance matrix estimated with and without ouliers.

Robustness of ANMF: Impact on detection performance

Same target $\mathbf{y}_k = \mathbf{p}_0$ (SNR 20dB) than those in the cell under test in the reference cells (case of convoy for example)



AMF + SCM



ANMF + FPE

The SCM can whiten the target to detect,
The ANMF built with FPE is more robust.

Outline

1. Adaptive Robust Detection Schemes in non-Gaussian Background
   - CES distributions
   - M-estimators and Tyler (FP) Estimator
   - Robustness of M-estimators and ANMF
   - MUltiple Signal Classification (MUSIC) method
2. Other Refinements
   - Exploiting Prior Information: Covariance Structure
   - Low Rank Detectors
   - Shrinkage of M-estimator
   - RMT Theory and M-Estimator based Detectors

MUltiple Signal Classification (MUSIC) method

- K (known) direction of arrival θ_k on m antennas
- Gaussian stationary narrowband signal with additive noise.
- the DoA [Schmidt, 1986] is estimated from n snapshots, using the SCM, the Huber's M-estimator and the Tyler's estimator.

$$\mathbf{y}(t) = \mathbf{A}(\theta_0)\,\mathbf{s}(t) + \mathbf{w}(t)\,.$$

- $\theta_0 = (\theta_1, \theta_2, \ldots, \theta_K)^T$,
- the steering matrix $\mathbf{A}(\theta_0) = (\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K))$,
- $\mathbf{s}(t) = (s_1(t), s_2(t), \ldots, s_K(t))^T$ signal vector,
- $\mathbf{w}(t)$ stationary additive noise.

$$\Sigma = \mathbb{E}[\mathbf{y}\,\mathbf{y}^H] = \mathbf{A}(\theta_0)\,\mathbb{E}[\mathbf{s}\mathbf{s}^H]\,\mathbf{A}^H(\theta_0) + \sigma^2\mathbf{I}\,.$$

which can be rewritten

$$\Sigma = \mathbb{E}[\mathbf{y}\,\mathbf{y}^H] = \mathbf{E}_S\,\mathbf{D}_S\,\mathbf{E}_S^H + \sigma^2\,\mathbf{E}_W\,\mathbf{E}_W^H\,,$$

where $\mathbf{E}_S$ (resp. $\mathbf{E}_W$) are the signal (resp. noise) subspace eigenvectors. The MUSIC statistic is

$$\begin{cases} H(\Sigma) = \underset{\theta}{\operatorname{argmax}}\,\gamma(\theta) \quad \text{where } \gamma(\theta) = \mathbf{s}(\theta)^H\,\mathbf{E}_W\,\mathbf{E}_W^H\,\mathbf{s}(\theta), \\ H(\widehat{\Sigma}) = \underset{\theta}{\operatorname{argmax}}\,\hat{\gamma}(\theta) \quad \text{where } \hat{\gamma}(\theta) = \displaystyle\sum_{i=1}^{m-K}\mathbf{s}(\theta)^H\,\hat{\mathbf{e}}_i\,\hat{\mathbf{e}}_i^H\,\mathbf{s}(\theta)\,, \end{cases}$$

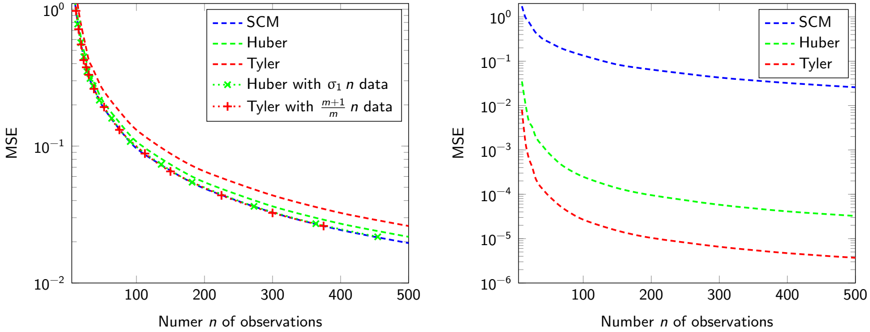
where $\hat{\mathbf{e}}_i$ are the eigenvectors of $\widehat{\Sigma}$.

This function respects assumptions of theorem 2!

Simulation using the MUltiple Signal Classification (MUSIC) method

The Mean Square Error (MSE) between the estimated angle $\widehat{\theta}$ and the real angle θ can then computed (case of one source).

- A $m = 3$ uniform linear array (ULA) with half wavelength sensors spacing is used,
- Gaussian stationary narrowband signal with DoA 20° plus additive noise.
- the DoA is estimated from n snapshots, using the SCM, the Huber's M-estimator and the Tyler's estimator.



(a) White additive Gaussian noise (b) K-distributed additive noise ($\nu = 0.1$)

Figure: MSE of $\hat{\theta}$ vs the number n of observations, with $m = 3$.

Similar conclusions as for detection can be drawn...

Outline

Motivations

The estimation of Σ does not take into account any prior knowledge on the covariance matrix:

How to improve detection performance by exploiting prior information on Σ ?

$\Longrightarrow$ Use of some prior knowledge on the structure of the covariance matrix:

- Toeplitz: [Burg et al., 1982] for estimation,
- known rank $r < m$ (ex: subspace detector),
- Persymmetry: [Nitzberg and Burke, 1980] for estimation, [Cai and Wang, 1992] for detection in Gaussian case, [Conte and Maio, 2003, Pailloux et al., 2011] in non-Gaussian noise.

Using Persymmetry Property

Under persymmetric considerations (ex: symmetrically spaced linear array, symmetrically spaced pulse train, ...), the Hermitian covariance matrix Σ verifies: $\Sigma = \mathbf{J}_m \, \Sigma^* \, \mathbf{J}_m$, where $\mathbf{J}_m$ is the m-dimensional antidiagonal matrix having 1 as non-zero elements. If the unitary matrix $\mathbf{T}$ is defined by:

$$\mathbf{T} = \begin{cases} \dfrac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{m/2} & \mathbf{J}_{m/2} \\ i\,\mathbf{I}_{m/2} & -i\,\mathbf{J}_{m/2} \end{pmatrix} & \text{for } m \text{ even} \\[2ex] \dfrac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{(m-1)/2} & 0 & \mathbf{J}_{(m-1)/2} \\ 0 & \sqrt{2} & 0 \\ i\,\mathbf{I}_{(m-1)/2} & 0 & -i\,\mathbf{J}_{(m-1)/2} \end{pmatrix} & \text{for } m \text{ odd}\,, \end{cases} \tag{9}$$

then:

- $\mathbf{s} = \mathbf{T}\,\mathbf{p}$ is a real vector (if $\mathbf{p}$ is centrosymmetric, i.e. $\mathbf{p} = \mathbf{J}_m\,\mathbf{p}^*$),
- $\mathbf{R} = \mathbf{T}\,\Sigma\,\mathbf{T}^H$ is a real symmetric matrix.

Equivalent Detection Problem

Using previous transformation $\mathbf{T}$, the original problem can be reformulated as:

Original Problem	$\mathbf{T}$	**Equivalent Problem**
$\begin{cases} H_0 : \mathbf{y} = \mathbf{c}, & \mathbf{c}_1, \ldots, \mathbf{c}_n \\ H_1 : \mathbf{y} = A\,\mathbf{p} + \mathbf{c}, & \mathbf{c}_1, \ldots, \mathbf{c}_n \end{cases}$	$\rightarrow$	$\begin{cases} H_0 : \mathbf{z} = \mathbf{n}, & \mathbf{n}_1, \ldots, \mathbf{n}_n \\ H_1 : \mathbf{z} = A\,\mathbf{s} + \mathbf{n}, & \mathbf{n}_1, \ldots, \mathbf{n}_n \end{cases}$

where

- $\mathbf{z} = \mathbf{T}\,\mathbf{y} \in \mathbb{C}^m$,
- $\mathbf{n} = \sqrt{\tau}\,\mathbf{x}$ and $\mathbf{n}_k = \sqrt{\tau_k}\,\mathbf{x}_k$ with $\mathbf{x}, \mathbf{x}_k \sim \mathcal{CN}(\mathbf{0}_m, \mathbf{R})$ where $\mathbf{R}$ is an unknown real symmetric matrix,
- $\mathbf{s} = \mathbf{T}\,\mathbf{p}$ is a real vector.

The main motivation for introducing the transformed data is that the original persymmetric complex covariance matrix of the Gaussian speckle Σ is transformed though $\mathbf{T}$ onto a real covariance matrix $\mathbf{R}$.

The Persymmetric FP Covariance Matrix Estimate

From the estimate $\widehat{\mathbf{R}}_{FP}$ of the real covariance matrix $\mathbf{R}$, solution of the following equation:

$$\widehat{\mathbf{R}} = \frac{m}{n} \sum_{k=1}^{n} \frac{\mathbf{n}_k \, \mathbf{n}_k^H}{\mathbf{n}_k^H \, \widehat{\mathbf{R}}^{-1} \, \mathbf{n}_k} \,,$$

the Persymmetric Fixed-Point Covariance Matrix Estimate can be defined as:

$$\boxed{\widehat{\mathbf{R}}_{PFP} = \mathcal{R}e(\widehat{\mathbf{R}}_{FP}).}$$

Statistical performance of $\widehat{\mathbf{R}}_{PFP}$ [Pailloux et al., 2008, Pailloux et al., 2011]:

- $\widehat{\mathbf{R}}_{PFP}$ is a consistent estimate of $\mathbf{R}$ when n tends to infinity,
- $\widehat{\mathbf{R}}_{PFP}$ is an unbiased estimate of $\mathbf{R}$,
- Its asymptotic distribution is the same as the asymptotic distribution of a real Wishart matrix with $\dfrac{m}{m+1} \, 2\, n$ degrees of freedom.

The Persymmetric Adaptive Normalized Matched Filter

The resulting P-ANMF for the transformed problem is based on the PFP estimate and can be defined as:

$$\Lambda(\widehat{\mathbf{R}}_{PFP}) = \frac{|\mathbf{s}^\top \widehat{\mathbf{R}}_{PFP}^{-1} \mathbf{z}|^2}{(\mathbf{s}^\top \widehat{\mathbf{R}}_{PFP}^{-1} \mathbf{s})(\mathbf{z}^H \widehat{\mathbf{R}}_{PFP}^{-1} \mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda. \tag{10}$$

Properties:

- $\Lambda(\widehat{\mathbf{R}}_{PFP})$ is texture-CFAR,
- $\Lambda(\widehat{\mathbf{R}}_{PFP})$ is matrix-CFAR,
- The use of PFP estimate in the ANMF allows to virtually double the number n of secondary data and improve the performance of the ANMF detector built with the FP matrix estimate.

$\boxed{\Lambda(\widehat{\mathbf{R}}_{PFP}) \text{ is SIRV-CFAR and is called the P-ANMF.}}$

Statistical study of the P-ANMF

The analytical expression for the Probability Density Function of the test statistic $\Lambda(\widehat{\mathbf{R}}_{PFP})$ is really not easy to derive in a closed form but the following results gives some insight about its distribution.

$\Lambda(\widehat{\mathbf{R}}_{PFP})$ has the same distribution as $\dfrac{F}{F+1}$ where

$$F = \frac{(\alpha_1\, u_{22} - \alpha_2\, u_{21})^2 + \left(1 + \left(\dfrac{\beta_3}{u_{33}}\right)^2\right)(a\, u_{22} - b\, u_{21})^2}{(\alpha_2\, u_{11})^2 + \left(t_{11}\, u_{22}\, \dfrac{\beta_3}{u_{33}}\right)^2 + u_{11}^2 \left(1 + \left(\dfrac{\beta_3}{u_{33}}\right)^2\right) b^2} \tag{11}$$

and where: a, b, α_1, $u_{21} \sim \mathcal{N}(0,1)$, $\alpha_2^2 \sim \chi^2_{m-1}$, $\beta_3^2 \sim \chi^2_{m-2}$, $u_{11}^2 \sim \chi^2_{n'-m+1}$, $u_{22}^2 \sim \chi^2_{n'-m+2}$, $u_{33}^2 \sim \chi^2_{n'-m+3}$ with $n' = \dfrac{m}{m+1}\, 2\, n$.

Outline

Conventional Low Rank Detectors

Principle of Low Rank Matched Filter approaches found for example in [Kirsteins and Tufts, 1994] (Principal Component Inverse) and [Haimovich, 1996] (Eigencanceler) and [Rangaswamy et al., 2004].

Let suppose the rank r of clutter covariance matrix Σ is known:

- Example of sidelooking STAP with M pulses measurements and N sensors, $r = N + (M-1)\,\beta$ (Brennan's rule) where $\beta = 2\,v\,T_r/d$.

The idea is to **project the data onto the orthogonal subspace of the clutter**.

$$\hat{\Sigma}_n = \frac{1}{n}\sum_{k=1}^{n} \mathbf{y}_k\,\mathbf{y}_k^H = (\mathbf{U_r}\,\mathbf{U_0}) \begin{pmatrix} \Sigma_r & \mathbf{0} \\ \mathbf{0} & \Sigma_0 \end{pmatrix} (\mathbf{U_r}\,\mathbf{U_0})^H,$$

If we denote by $\mathbf{\Pi}_{SCM} = \mathbf{U}_r\,\mathbf{U}_r^H$ the projector onto the clutter subspace, the Low-Rank ANMF detector is given by:

$$\Lambda_{LR-ANMF-SCM}(\mathbf{z}) = \frac{|\mathbf{p}^H\,(\mathbf{I}-\mathbf{\Pi}_{SCM})\,\mathbf{z}|^2}{(\mathbf{p}^H\,(\mathbf{I}-\mathbf{\Pi}_{SCM})\,\mathbf{p})(\mathbf{z}^H\,(\mathbf{I}-\mathbf{\Pi}_{SCM})\,\mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda.$$

Extended Low Rank Detectors

In case of heterogeneous and non-Gaussian clutter, we know that $\hat{\Sigma}_{SCM}$ or Π_{SCM} are not good estimates. If we denote the Normalized Sample Covariance Matrix by:

$$\Sigma_{NSCM} = \frac{N\,M}{n} \sum_{k=1}^{n} \frac{\mathbf{y}_k\,\mathbf{y}_k^H}{\mathbf{y}_k^H\,\mathbf{y}_k} = (\mathbf{U_r}\,\mathbf{U_0}) \begin{pmatrix} \Sigma_r & \mathbf{0} \\ \mathbf{0} & \Sigma_0 \end{pmatrix} (\mathbf{U_r}\,\mathbf{U_0})^H$$

[Ginolhac et al., 2012, Ginolhac et al., 2013] proved that $\mathbf{\Pi}_{NSCM} = \mathbf{U}_r\,\mathbf{U}_r^H$ is a consistent estimate projector onto the clutter subspace. We can define the extended Low-Rank ANMF-NSCM:

$$\Lambda_{LR-ANMF-NSCM}(\mathbf{y}) = \frac{|\mathbf{p}^H\,(\mathbf{I}-\mathbf{\Pi}_{NSCM})\,\mathbf{z}|^2}{(\mathbf{p}^H\,(\mathbf{I}-\mathbf{\Pi}_{NSCM})\,\mathbf{p})(\mathbf{z}^H\,(\mathbf{I}-\mathbf{\Pi}_{NSCM})\,\mathbf{z})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda.$$

This detector is found to be **texture-CFAR** and is **asymptotically** Σ**-CFAR**. Moreover, he has another nice **robustness property** when outliers and targets are present in the secondary data. The Normalized Sample Covariance Matrix is a good candidate for adaptive version of Rangaswami's Low Rank Matched Filter and Low Rank Normalized Matched Filter.

More recent works can be found in
[Breloy et al., 2015, Sun et al., 2016, Breloy et al., 2016, Ginolhac and Forster, 2016].

Outline

Shrinkage of Tyler's estimators

Case of small number of observations or under-sampling $n < m$: matrix is not invertible
$\Rightarrow$ Problem when using M-estimators or Tyler's estimator!

Chen estimator

$$\widehat{\Sigma}_C = (1-\beta)\,\frac{m}{n}\sum_{i=1}^{n}\frac{\mathbf{z}_i\mathbf{z}_i^H}{\mathbf{z}_i^H\widehat{\Sigma}_C^{-1}\mathbf{z}_i} + \beta\mathbf{I}$$

subject to the constraint $\mathrm{Tr}(\widehat{\Sigma}) = m$ and for $\beta \in (0,1]$.

- Originally introduced in [Abramovich and Spencer, 2007],
- Existence, uniqueness and algorithm convergence proved in [Chen et al., 2011],
- Active research [Abramovich and Besson, 2013, Besson and Abramovich, 2013], R. Couillet, M. McKay, A. Wiesel, F. Pascal.

Shrinkage Tyler's estimators

Pascal estimator [Pascal et al., 2014]

$$\widehat{\Sigma}_P = (1-\beta)\,\frac{m}{n}\sum_{i=1}^{n}\frac{\mathbf{z}_i\mathbf{z}_i^H}{\mathbf{z}_i^H\widehat{\Sigma}_P^{-1}\mathbf{z}_i} + \beta\mathbf{I}$$

subject to the **no** trace constraint but for $\beta \in (\bar{\beta}, 1]$, where $\bar{\beta} := \max(0, 1-n/m)$.

- $\widehat{\Sigma}_P$ (naturally) verifies $\mathrm{Tr}(\widehat{\Sigma}_P^{-1}) = m$ for all $\beta \in (0,1]$,
- Existence, uniqueness and algorithm convergence proved,
- The main challenge is to find the optimal β! [Couillet and McKay, 2014].

Outline

Radar Detection Schemes for Joint Time and Spatial Correlated Clutter

Motivations: Adaptive radar detection and estimation schemes are often based on the independence of the secondary data used for building estimators and detectors. This independence allows to build Likelihood functions.

Example: estimating a covariance matrix $\mathbf{M}$

With a given set of n independent m-dimensional vectors $\{\mathbf{y}_i\}_{i\in[1,n]}$ distributed according to $\mathcal{CN}(\mathbf{0}_m, \mathbf{M})$, the corresponding Likelihood function Λ can be built as

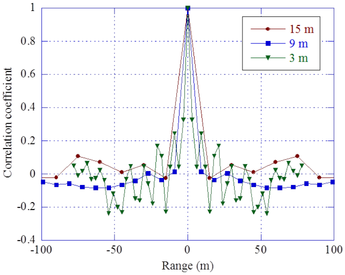
$$\Lambda(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n \,|\, \mathbf{M}) = \prod_{i=1}^{n} p(\mathbf{y}_i) = \prod_{i=1}^{n} \frac{1}{\pi^m |\mathbf{M}|} \exp\left(-\mathbf{y}_i^H \mathbf{M}^{-1} \mathbf{y}_i\right).$$

The Maximum Likelihood Estimate $\widehat{\mathbf{M}}$ of $\mathbf{M}$ is the zero of the partial derivative of $\Lambda(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n \,|\, \mathbf{M})$ with respect to $\mathbf{M}$ leading to the well known SCM.

Motivations

In many radar and imagery applications, data $\{\mathbf{y}_i\}_{i\in[1,n]}$ can be viewed as a joint spatial and temporal process:

- For high resolution radar, the sea clutter is clearly jointly spatially and temporally correlated,



Sea clutter spatial correlation, IPIX radar [Greco et al., 2006].

Motivations

- In mutichannel (polarimetric, interferometric or multi-temporal) SAR imaging, the multivariate vector characterizing each spatial pixel of the image is correlated over the channels but can also be strongly correlated with those of neighbourhood pixels,
- When a radar signal with bandwidth B is oversampled ($Fe = k\,B$, $k > 1$), the associated range bins can be spatially correlated and the measurements are not independent anymore.

In the radar community, one generally supposes that the vectors of information collected over a spatial support are identically and independently distributed.

This problem could be, for example, adressed using Multidimensional Space-time ARMA modeling.

The aim of this work is to relax this hypothesis through the use of recent Random Matrix Theory results.

Problem formulation

Detection of a complex signal corrupted by an additive Gaussian noise $\mathbf{c} \sim \mathcal{CN}(\mathbf{0}_m, \mathbf{M})$ in a N-dimensional complex observation vector $\mathbf{y}$:

$$\begin{cases} H_0 : \mathbf{y} = \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, n \\ H_1 : \mathbf{y} = \alpha\, \mathbf{p} + \mathbf{c} & \mathbf{y}_i = \mathbf{c}_i \quad i = 1, \ldots, n \end{cases},$$

where $\mathbf{p}$ is a perfectly known complex steering vector, α is the unknown signal amplitude and where the $\mathbf{c}_i \sim \mathcal{CN}(\mathbf{0}_m, \mathbf{M})$ are n signal-free non independent measurements. The covariance matrix $\mathbf{M}$ characterizes the temporal or spectral correlation within the components of the noise vectors.

To model the spatial dependency between the secondary data, from the Gaussian assumption on $\mathbf{c}_i$, we may write the $m \times n$-matrix $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_n]$ under the following form:

$$\mathbf{C} = \mathbf{M}^{1/2}\, \mathbf{X} \mathbf{T}^{1/2},$$

where $\mathbf{M} \in \mathbb{C}^{m \times m}$ and $\mathbf{T} \in \mathbb{C}^{m \times n}$ are both nonnegative definite, $\mathbf{X}$ is standard Gaussian $\mathcal{CN}(\mathbf{0}_m, \mathbf{I}_m)$, and where $\mathbf{T}$ satisfies the normalization $\frac{1}{n}\,\mathrm{tr}(\mathbf{T}) = 1$.

Problem formulation

The matrix $\mathbf{T}$ is considered Toeplitz, i.e., for all i, j, $\mathbf{T}_{i,j} = t_{|i-j|}$ for $t_0 = 1$ and $t_k \in \mathbb{C}$, and positive definite. Besides, $\sum_{k=0}^{n-1} |t_k| < \infty$.

Example: $m = 2$, $n = 3$

$$\mathbf{C} = \underbrace{\underbrace{\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}^{1/2}}_{\text{Temporal correlation}} \begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix}}_{\text{Temporal or Spectral Measurements}} \underbrace{\begin{pmatrix} t_0 & t_1 & t_2 \\ t_1 & t_0 & t_1 \\ t_2 & t_1 & t_0 \end{pmatrix}^{1/2}}_{\text{Spatial correlation}}.$$

Some RMT results

Proposition: Consistent Estimation for $\mathbf{T}$ [Couillet et al., 2015]

As $m, n \to \infty$ such that $m/n \to c \in [0, \infty[$, and for every $\beta < 1$,

$$m^{\beta} \left\| \mathcal{T}\left[\frac{1}{m}\,\mathbf{C}^H\,\mathbf{C}\right] - \left(\frac{1}{m}\operatorname{tr}\,\mathbf{M}\right)\,\mathbf{T}\right\|_F \xrightarrow{a.s.} 0\,,$$

where $\mathcal{T}[\cdot]$ is the Toeplitzification operator: $(\mathcal{T}[\mathbf{X}])_{ij} = \dfrac{1}{n}\displaystyle\sum_{k=1}^{n} \mathbf{X}_{k,k+|i-j|}$.

Up to a constant, a consistent estimator $\hat{\mathbf{T}}$ of the spatial covariance $\mathbf{T}$ characterizing data $\{\mathbf{c}_i\}_{i\in[1,n]}$ is therefore defined as $\hat{\mathbf{T}} \propto \mathcal{T}\left[\frac{1}{m}\,\mathbf{C}^H\,\mathbf{C}\right]$ and the associated time whitened sample covariance matrix estimate $\widehat{\mathbf{M}}$ of $\mathbf{M}$ is defined as $\widehat{\mathbf{M}} \propto \frac{1}{n}\,\mathbf{C}\,\hat{\mathbf{T}}^{-1}\,\mathbf{C}^H$.

This technique has been extended in the framework of robust M-estimators.

Gaussian and non-Gaussian scenarios

Simulated Data: joint spatial and time correlated Gaussian or K-distributed ($\nu = 0.5$) data characterized by $m = 10$ pulses, $n = 20$ secondary data where:

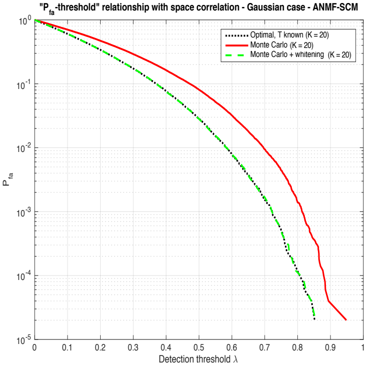
$$\mathbf{M} = \left(\rho_{\mathbf{M}}^{|i-j|}\right)_{i,j\in[1,m]}, \ \mathbf{T} = \left(\rho_{\mathbf{T}}^{|i-j|}\right)_{i,j\in[1,n]} \text{ with } \rho_{\mathbf{M}} = 0.5, \ \rho_{\mathbf{T}} = 0.9.$$

To evaluate the detection performance of the Λ_{ANMF} test statistic, we have compared three approaches:

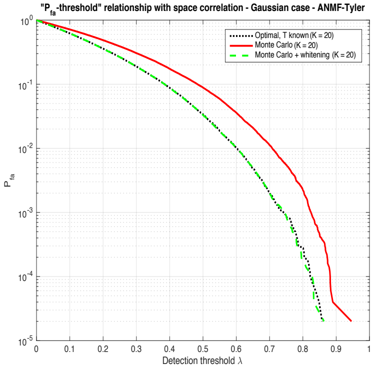
- $\mathbf{M}$ is unknown but $\mathbf{T}$ is assumed to be known: the covariance estimate $\widehat{\mathbf{M}}$ is either given by $\frac{1}{n}\mathbf{C}\,\mathbf{T}^{-1}\,\mathbf{C}^H$ (SCM) or the Tyler's estimate of the true spatial-whitened data $\mathbf{C}\,\mathbf{T}^{-1/2}$,
- $\mathbf{T}$ is assumed to be unknown and is estimated through $\hat{\mathbf{T}} \propto \mathcal{T}\left[\frac{1}{m}\,\mathbf{C}^H\,\mathbf{C}\right]$: the covariance estimate $\widehat{\mathbf{M}}$ is either given by $\frac{1}{n}\,\mathbf{C}\,\hat{\mathbf{T}}^{-1}\,\mathbf{C}^H$ (SCM) or the Tyler's estimate of the spatial-whitened data $\mathbf{C}\,\hat{\mathbf{T}}^{-1/2}$,
- the classical approach that does not take into account the space correlation: the covariance estimate $\widehat{\mathbf{M}}$ is either given by $\frac{1}{n}\mathbf{C}\,\mathbf{C}^H$ (SCM) or Tyler's estimate of the data $\mathbf{C}$.

False Alarm Regulation - Gaussian Case

ANMF-SCM



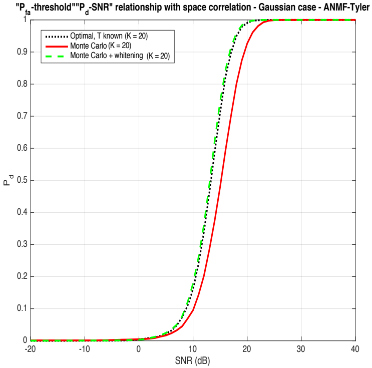
ANMF-Tyler



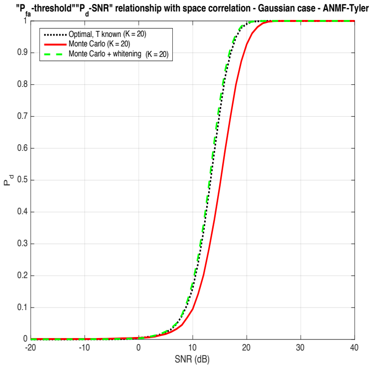
Same False Alarm Regulation performance for ANMF-SCM and ANMF-Tyler (Gaussian case)

Associated Detection Performance - Gaussian Case

ANMF-SCM



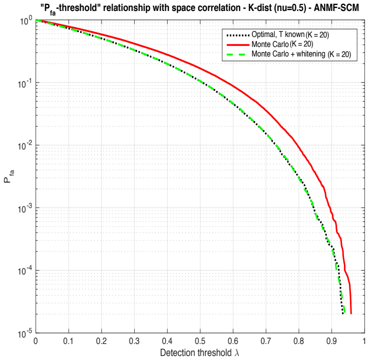
ANMF-Tyler



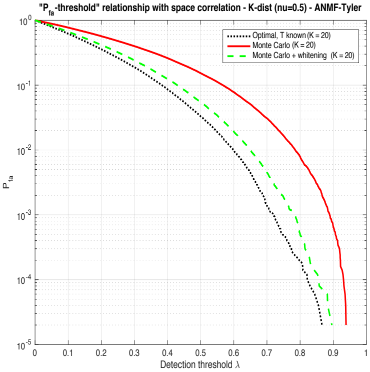
- Same Probability of Detection performance.
- Around 3dB gain improvement with RMT whitening procedure

False Alarm Regulation - K-distributed Case

ANMF-SCM



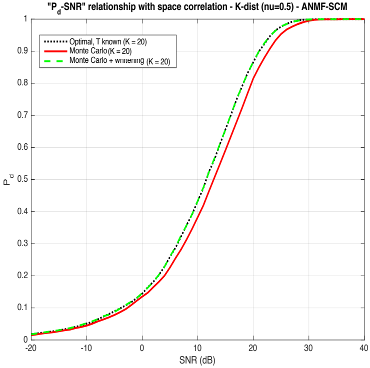
ANMF-Tyler



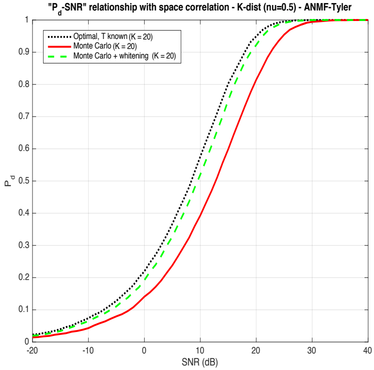
- Better False Alarm regulation performance for ANMF-FP (Non-Gaussian case).
- Better False Alarm regulation with RMT whitening procedure

Associated Detection Performance - K-distributed Case

ANMF-SCM



ANMF-Tyler



- Better performances in terms of Probability of Detection performance for ANMF-Tyler.
  - Around 3dB gain improvement with RMT whitening procedure

End of Part B

Questions?

References I

Abramovich, Y. I. and Besson, O. (2013).
Regularized covariance matrix estimation in complex elliptically symmetric distributions using the expected likelihood approach - part 1: The over-sampled case.
Signal Processing, IEEE Transactions on, 61(23):5807–5818.

Abramovich, Y. I. and Spencer, N. K. (2007).
Diagonally loaded normalised sample matrix inversion (lnsmi) for outlier-resistant adaptive filtering.
In *2007 IEEE International Conference on Acoustics, Speech and Signal Processing - ICASSP '07*, volume 3, pages III–1105–III–1108.

Bausson, S., Pascal, F., Forster, P., Ovarlez, J.-P., and Larzabal, P. (2007).
First and second order moments of the normalized sample covariance matrix of spherically invariant random vectors.
IEEE SP Letters, 14(6):425–428.

Besson, O. and Abramovich, Y. I. (2013).
Regularized covariance matrix estimation in complex elliptically symmetric distributions using the expected likelihood approach - part 2: The under-sampled case.
Signal Processing, IEEE Transactions on, 61(23):5819–5829.

Breloy, A., Ginolhac, G., Pascal, F., and Forster, P. (2015).
Clutter subspace estimation in low rank heterogeneous noise context.
Signal Processing, IEEE Transactions on, 63(9):2173–2182.

References II

Breloy, A., Ginolhac, G., Pascal, F., and Forster, P. (2016).
Robust covariance matrix estimation in heterogeneous low rank context.
Signal Processing, IEEE Transactions on, 64(22):5794–5806.

Burg, J. P., Luenberger, D. G., and Wenger, D. L. (1982).
Estimation of structured covariance matrices.
Proc. IEEE, 70(9):963–974.

Cai, L. and Wang, H. (1992).
A Persymmetric Multiband GLR Algorithm.
Aerospace and Electronic Systems, IEEE Transactions on, pages 806–816.

Chen, Y., Wiesel, A., and Hero, A. O. (2011).
Robust shrinkage estimation of high-dimensional covariance matrices.
Signal Processing, IEEE Transactions on, 59(9):4097–4107.

Conte, E. and Maio, A. D. (2003).
Exploiting Persymmetry for CFAR Detection in Compound-Gaussian Clutter.
IEEE Trans. on AES, 39:719–724.

Couillet, R., Greco, M. S., Ovarlez, J. P., and Pascal, F. (2015).
RMT for whitening space correlation and applications to radar detection.
In *2015 IEEE 6th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, pages 149–152.

References III

Couillet, R. and McKay, M. (2014).
Large dimensional analysis and optimization of robust shrinkage covariance matrix estimators.
Journal of Multivariate Analysis, 131:99 – 120.

E. Conte and M. Lops and G. Ricci (1995).
Asymptotically Optimum Radar Detection in Compound-Gaussian Clutter.
Aerospace and Electronic Systems, IEEE Transactions on, 31(2):617–625.

Frahm, G. (2004).
Generalized elliptical distributions: theory and applications.
PhD thesis, Universitätsbibliothek.

Ginolhac, G. and Forster, P. (2016).
Approximate distribution of the low-rank adaptive normalized matched filter test statistic under the null hypothesis.
Aerospace and Electronic Systems, IEEE Transactions on, 52(4):2016–2023.

Ginolhac, G., Forster, P., Pascal, F., and Ovarlez, J. P. (2012).
Derivation of the bias of the normalized sample covariance matrix in a heterogeneous noise with application to low rank STAP filter.
Signal Processing, IEEE Transactions on, 60(1):514–518.

Ginolhac, G., Forster, P., Pascal, F., and Ovarlez, J. P. (2013).
Performance of two low-rank STAP filters in a heterogeneous noise.
Signal Processing, IEEE Transactions on, 61(1):57–61.

References IV

Greco, M., Gini, F., and Rangaswamy, M. (2006).
Statistical analysis of measured polarimetric clutter data at different range resolutions.
IEE Proceedings - Radar, Sonar and Navigation, 153(6):473–481.

Haimovich, A. (1996).
The eigencanceler: adaptive radar by eigenanalysis methods.
Aerospace and Electronic Systems, IEEE Transactions on, 32(2):532–542.

Huber, P. J. (1964).
Robust estimation of a location parameter.
The Annals of Mathematical Statistics, 35(1):73–101.

Jay, E. (2002).
Detection in non-Gaussian noise.
PhD thesis, University of Cergy-Pontoise, France.

Kelker, D. (1970).
Distribution theory of spherical distributions and a location-scale parameter generalization.
Sankhya: The Indian Journal of Statistics, Series A, 32(4):419–430.

Kent, J. T. and Tyler, D. E. (1991).
Redescending M-estimates of multivariate location and scatter.
Annals of Statistics, 19(4):2102–2119.

References V

Kirsteins, I. P. and Tufts, D. W. (1994).
Adaptive detection using low rank approximation to a data matrix.
Aerospace and Electronic Systems, IEEE Transactions on, 30(1):55–67.

Kraut, S. and Scharf, L. (1999).
The CFAR adaptive subspace detector is a scale-invariant GLRT.
Signal Processing, IEEE Transactions on, 47(9):2538–2541.

Mahot, M. (2012).
Robust Covariance Estimation in Signal Processing.
PhD thesis, Ecole Normale de Cachan.

Mahot, M., Pascal, F., Forster, P., and Ovarlez, J. P. (2013).
Asymptotic properties of robust complex covariance matrix estimates.
Signal Processing, IEEE Transactions on, 61(13):3348–3356.

Maronna, R. A., Martin, D. R., and Yohai, J. V. (2006).
Robust Statistics: Theory and Methods.
Wiley Series in Probability and Statistics. John Wiley & Sons.

Nitzberg, R. and Burke, J. R. (1980).
Application of Maximum Likelihood estimation of persymmetric covariance matrices to adaptive detection.
Aerospace and Electronic Systems, IEEE Transactions on, 25:124–127.

References VI

Ollila, E., Tyler, D. E., Koivunen, V., and Poor, H. V. (2012).
Complex elliptically symmetric distributions: Survey, new results and applications.
Signal Processing, IEEE Transactions on, 60(11):5597 –5625.

Ovarlez, J. P., Pascal, F., and Breloy, A. (2015).
Asymptotic detection performance analysis of the robust adaptive normalized matched filter.
In *2015 IEEE 6th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, pages 137–140.

Pailloux, G., Forster, P., Ovarlez, J.-P., and Pascal, F. (2008).
On Persymmetric Covariance Matrices in Adaptive Detection.
IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP-08, pages 2305–2308.

Pailloux, G., Forster, P., Ovarlez, J.-P., and Pascal, F. (2011).
Persymmetric adaptive radar detectors.
Aerospace and Electronic Systems, IEEE Transactions on, 47(4):2376–2390.

Pascal, F. (2006).
Détection et Estimation en Environnement Non-Gaussien.
PhD thesis, University of Nanterre / ONERA, France.

Pascal, F., Chitour, Y., Ovarlez, J. P., Forster, P., and Larzabal, P. (2008a).
Covariance structure maximum-likelihood estimates in compound gaussian noise: Existence and algorithm analysis.
Signal Processing, IEEE Transactions on, 56(1):34–48.

References VII

Pascal, F., Chitour, Y., and Quek, Y. (2014).
Generalized robust shrinkage estimator and its application to STAP detection problem.
Signal Processing, IEEE Transactions on, 62(21):5640–5651.

Pascal, F., Forster, P., Ovarlez, J. P., and Larzabal, P. (2008b).
Performance analysis of covariance matrix estimates in impulsive noise.
Signal Processing, IEEE Transactions on, 56(6):2206–2217.

Pascal, F. and Ovarlez, J. P. (2015).
Asymptotic properties of the robust ANMF.
In *2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 2594–2598.

Pascal, F., Ovarlez, J.-P., Forster, P., and Larzabal, P. (2004).
Constant false alarm rate detection in spherically invariant random processes.
In *Proc. of the European Signal Processing Conf., EUSIPCO-04*, pages 2143–2146, Vienna.

Rangaswamy, M., Lin, F. C., and Gerlach, K. R. (2004).
Robust adaptive signal processing methods for heterogeneous radar clutter scenarios.
Signal Processing, 84(9):1653 – 1665.
Special Section on New Trends and Findings in Antenna Array Processing for Radar.

Schmidt, R. O. (1986).
Multiple emitter location and signal parameter estimation.
Acoustics Speech and Signal Processing, IEEE Transactions on, 34(3):276–280.

References VIII

Sun, Y., Breloy, A., Babu, P., Palomar, D. P., Pascal, F., and Ginolhac, G. (2016).
Low-complexity algorithms for low rank clutter parameters estimation in radar systems.
Signal Processing, IEEE Transactions on, 64(8):1986–1998.

Tyler, D. E. (1987).
A distribution-free M-estimator of multivariate scatter.
The Annals of Statistics, 15(1):234–251.

Yao, K. (1973).
A Representation Theorem and its Applications to Spherically Invariant Random Processes.
Information Theory, IEEE Transactions on, 19:600–608.