EXPLOITING PERSYMMETRY FOR LOW-RANK SPACE TIME ADAPTIVE PROCESSING

G. Ginolhac[1], *P. Forster*[1], *F. Pascal*[2] *and J. P. Ovarlez*[2,3]

[1] SATIE, ENS Cachan, CNRS, UniverSud, 61, av President Wilson, F-94230 Cachan, France
[2] SONDRA, Supelec, Plateau du Moulon, 3 rue Joliot-Curie, F-91192 Gif-sur-Yvette CEDEX, France
[3] ONERA, DEMR/TSI, Chemin de la Hunière, F-91120 Palaiseau, France

ABSTRACT

Reducing the number of secondary data used to estimate the Covariance Matrix (CM) for Space Time Adaptive Processing (STAP) techniques is still an active research topic. Within this framework, the Low-Rank (LR) structure of the clutter is well-known and the corresponding LR STAP filters have been shown to exhibit a smaller SNR loss than classical STAP filters, $2r$ secondary data (r is the clutter rank) instead of $2m$ (m is the data size) is needed to reach a 3dB SNR loss. By using other features of the radar system, other properties of the CM could be exploited to further reduce the number of secondary data: this is the case for active systems using a symmetrically spaced linear array with constant pulse repetition interval. In this context, we propose to combine the resulting persymmetric property of the CM and the LR structure of the clutter to perform the CM estimation. In this paper, the resulting STAP filter is shown, both theoretically and experimentally, to exhibit good performance with fewer secondary data: 3dB SNR loss is achieved with only r secondary data.

Index Terms**—** STAP, Low-Rank clutter, Persymmetry, Perturbation Analysis.

1. INTRODUCTION

In Space Time Adaptive Processing (STAP) for radar applications [1], the disturbance is composed of white Gaussian thermal noise plus ground clutter. In this paper, we assume that the ground clutter is heterogeneous and that it can no longer be modeled by a Gaussian process. To take this heterogeneity into account, one can use the Spherically Invariant Random Vector (SIRV) product model, first introduced by Yao [2]. Moreover, the ground clutter Covariance Matrix (CM) is known to possess only a few non-zero eigenvalues [3], resulting in a low-rank structure. This low rank-structure can be exploited for target detection by designing adaptive filters which require much less secondary data than conventional adaptive schemes with equivalent performance [4, 5]. These low-rank STAP filters require the estimation of the projector onto the clutter subspace, which itself requires the estimation of the CM. The latter is estimated from the secondary data which consist of noise plus clutter only. The Sample Covariance Matrix (SCM) is usually used for this purpose. Moreover, many applications can lead to a particular structure of the CM. Such a situation is frequently met in array processing and in particular in radar systems using a symmetrically spaced linear array for spatial domain processing, or symmetrically spaced pulse train for temporal domain processing [6, 7, 8]. In these systems, the CM has the persymmetric property. It is well known that this persymmetric structure could be exploited to improve the estimation quality. In particular, the persymmetric Maximum Likelihood Estimate (MLE) of the CM is used instead of the SCM [9, 10] to improve the performance of adaptive detectors. But in a Low-Rank context, this persymmetric structure is not often used in detectors or STAP filters.

We propose in this paper to build our projector onto the clutter subspace from this MLE which results in a new LR STAP filter. We also investigate the theoretical SNR Loss, based on a perturbation analysis [11], of this new LR STAP filter in SIRV clutter plus white Gaussian noise context, extending existing results available in the literature for the SCM in Gaussian clutter [5]. In particular, we show that the final result does not depend on the SIRV texture which implies that the SCM is sufficient to estimate the CM (normalization of secondary data is not needed). Simulations with simulated data and real data (real clutter STAP and synthetic targets) illustrate the theoretical result: this new STAP filter needs two times less secondary data than the classical LR STAP filter for equivalent performances.

The authors thank the DGA/MI for the STAP data.

2. LOW-RANK STAP FILTER

2.1. Signal Model

STAP [1] is applied to airborne radar in order to detect moving targets. Typically, the radar receiver consists in an array of N antenna elements processing M pulses in a coherent processing interval. In the following, let us set $m = NM$. In this framework, we assume that a known complex signal $\mathbf{d}$ corrupted by an additive disturbance $\mathbf{n}$ is in $\mathbf{x} \in \mathbb{C}^m$:

$$\mathbf{x} = \alpha \mathbf{d} + \mathbf{n}, \tag{1}$$

where α is a complex attenuation. We assume to have K secondary data $\mathbf{x}_k$ which only contain the disturbance:

$$\mathbf{x}_k = \mathbf{n}_k \quad k = 1, \ldots, K \tag{2}$$

We assume that $\mathbf{n}$ and $\mathbf{n}_k$ are independent and share the same statistical distribution and are modeled as the sum of a clutter, $\mathbf{c}$ or $\mathbf{c}_k$, and a white Gaussian noise, $\mathbf{b}$ or $\mathbf{b}_k$:

$$\begin{array}{rcl} \mathbf{n} & = & \mathbf{c} + \mathbf{b} \\ \mathbf{n}_k & = & \mathbf{c}_k + \mathbf{b}_k \quad k = 1, \ldots, K \end{array} \tag{3}$$

The processes $\mathbf{b}$ and $\mathbf{b}_k$ are modeled as a zero-mean complex Gaussian noise $\mathscr{CN}(\mathbf{0}, \lambda \mathbf{I}_m)$ ($\mathbf{I}_m$ is the identity matrix). Concerning the clutter $\mathbf{c}$ and $\mathbf{c}_k$, we consider that their power in each cell k and the cell under test is different. In such a situation, it is common to model this kind of clutter by a SIRV [12]. A SIRV is a non-homogeneous Gaussian random vector with random power: its randomness is induced by spatial variation in the radar backscattering. The SIRV [2] $\mathbf{c}$ (resp. $\mathbf{c}_k$) is the product of a positive random variable τ (resp. τ_k), called the *texture*, and a m-dimensional independent complex Gaussian vector $\mathscr{CN}(\mathbf{0}, \mathbf{C})$ $\mathbf{g}$ (resp. $\mathbf{g}_k$), called the *speckle*, with zero-mean and CM $\mathbf{C} = E(\mathbf{g}\mathbf{g}^H) = E(\mathbf{g}_k \mathbf{g}_k^H)$:

$$\begin{array}{rcl} \mathbf{c} & = & \sqrt{\tau}\mathbf{g} \\ \mathbf{c}_k & = & \sqrt{\tau}_k \mathbf{g}_k \quad k = 1, \ldots, K \end{array} \tag{4}$$

In classical STAP context, we are able to evaluate the clutter rank thanks to the Brennan's formula [13] which leads to a low rank structure for the STAP clutter $\mathbf{c}$ and $\mathbf{c}_k$, e.g. $\text{rank}\,(\mathbf{C}) = r \ll m$. The *speckle* CM, $\mathbf{C}$, can be thus decomposed as:

$$\mathbf{C} = \sum_{i=1}^{r} \lambda_i \mathbf{u}_i \mathbf{u}_i^H \tag{5}$$

where $\lambda_1 > \lambda_2 > \ldots > \lambda_r > \lambda_{r+1} = \ldots = \lambda_{NM} = 0$ are the eigenvalues of $\mathbf{C}$ and $\{\mathbf{u}_1, \ldots, \mathbf{u}_r, \mathbf{u}_{r+1}, \ldots, \mathbf{u}_{NM}\}$ are the associated eigenvectors. The CM of $\mathbf{n}$ and $\mathbf{n}_k$ is then given by:

$$\mathbf{\Sigma} = E[\tau]\mathbf{C} + \lambda \mathbf{I}_m \tag{6}$$

Many applications can result in a CM that exhibits some particular structure. For radar systems using a symmetrically spaced linear array for spatial domain processing, or symmetrically spaced pulse train for temporal domain processing [6, 7, 8], the CM $\mathbf{\Sigma}$ has the persymmetric property:

$$\mathbf{\Sigma} = \mathbf{J}_m \mathbf{\Sigma}^* \mathbf{J}_m, \tag{7}$$

where $\mathbf{J}_m$ is the m-dimensional antidiagonal matrix having 1 as non-zero elements. The signal vector is also persymmetric $\mathbf{d} = \mathbf{J}_m \mathbf{d}^*$. The persymmetric property is used by transforming the complex primary data (1) and secondary data (2) into real data. The persymmetric operation can be characterized by an unitary matrix $\mathbf{T}$ defined as:

$$\mathbf{T} = \begin{cases} \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{m/2} & \mathbf{J}_{m/2} \\ i\mathbf{I}_{m/2} & \mathbf{J}_{m/2} \end{pmatrix} & \text{for } m \text{ even} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{(m-1)/2} & 0 & \mathbf{J}_{(m-1)/2} \\ 0 & \sqrt{2} & 0 \\ i\mathbf{I}_{(m-1)/2} & 0 & i\mathbf{J}_{(m-1)/2} \end{pmatrix} & \text{for } m \text{ odd} \end{cases} \tag{8}$$

Let us introduce the transformed data by $\mathbf{T}$: $\mathbf{x}' = \mathbf{T}\mathbf{x}$, $\mathbf{x}'_k = \mathbf{T}\mathbf{x}_k$, $\mathbf{d}' = \mathbf{T}\mathbf{d}$, $\mathbf{c}' = \mathbf{T}\mathbf{c}$, $\mathbf{c}'_k = \mathbf{T}\mathbf{c}_k$, $\mathbf{b}' = \mathbf{T}\mathbf{b}$ and $\mathbf{b}'_k = \mathbf{T}\mathbf{b}_k$. The primary and the secondary data (1), (2) become after transformation by $\boldsymbol{T}$:

$$\begin{array}{rcl} \mathbf{x}' & = & \mathbf{d}' + \mathbf{c}' + \mathbf{b}' \\ \mathbf{x}'_k & = & \mathbf{c}'_k + \mathbf{b}'_k \qquad k = 1, \ldots, K \end{array} \tag{9}$$

The CM of data (9) is then $\mathbf{\Sigma}' = \mathbf{T}\mathbf{\Sigma}\mathbf{T}^H$. Its eigendecomposition is:

$$\begin{array}{rcl} \mathbf{\Sigma}' & = & \sum_{i=1}^{r} E[\tau]\lambda_i \mathbf{u}'_i \mathbf{u}'^H_i + \lambda \sum_{i=1}^{m} \mathbf{u}'_i \mathbf{u}'^H_i \\ & = & \mathbf{S}'_\Sigma + \lambda \mathbf{I}_m \end{array} \tag{10}$$

where $\{\mathbf{u}'_1, \ldots, \mathbf{u}'_r, \mathbf{u}'_{r+1}, \ldots, \mathbf{u}'_m\}$ are the eigenvectors of $\mathbf{\Sigma}'$. We notice that the matrix covariance rank is unchanged by the operator $\mathbf{T}$. Let us introduce the pseudo-inverse, $\mathbf{M}'$, of $\mathbf{S}'_\Sigma$:

$$\mathbf{M}' = \sum_{i=1}^{r} \frac{1}{E[\tau]\lambda_i} \mathbf{u}'_i \mathbf{u}'^H_i \tag{11}$$

We define the projector onto the clutter subspace $\mathbf{\Pi}'_c$ and the projector onto the orthogonal of the clutter subspace $\mathbf{\Pi}'^{\perp}_c$ [4, 5]:

$$\begin{array}{rcl} \mathbf{\Pi}'_c & = & \sum_{i=1}^{r} \mathbf{u}'_i \mathbf{u}'^H_i \\ \mathbf{\Pi}'^{\perp}_c & = & \mathbf{I}_m - \mathbf{\Pi}'_c \end{array} \tag{12}$$

2.2. Optimal and Sub-optimal STAP filters

The optimal STAP filter is known to be defined as [1]:

$$\mathbf{w}'_{opt} = \mathbf{\Sigma}'^{-1} \mathbf{d}', \tag{13}$$

whereas in low-rank assumption, it is expressed as [4, 5]:

$$\mathbf{w}'_{lropt} = \mathbf{\Pi}'^{\perp}_c \mathbf{d}' \tag{14}$$

In practical cases, since the CM $\mathbf{\Sigma}'$ (and therefore also $\mathbf{\Pi}'_c$) is unknown, it is necessary to estimate them from the secondary data $\mathbf{x}'_k$ (9).

This estimation is classically performed by using the SCM, but the persymmetric structure of $\mathbf{\Sigma}$ could be exploited to improve the estimation quality. The persymmetric MLE of the CM could be used instead of the SCM. In [9, 10], the MLE, denoted $\hat{\mathbf{R}}'$ is given by:

$$\hat{\mathbf{R}}' = \mathcal{R}e(\mathbf{T}\hat{\mathbf{R}}_{SCM}\mathbf{T}^H) \tag{15}$$

where $\hat{\mathbf{R}}_{SCM}$ is the SCM computed from (2). From the eigenvectors $\{\hat{\mathbf{u}}'_1, \ldots, \hat{\mathbf{u}}'_m\}$ of $\hat{\mathbf{R}}'$, the estimates of the projectors (onto the subspace clutter and its complement) by using the persymmetric structure of the CM are defined as [4, 5] :

$$\hat{\mathbf{\Pi}}'_c = \sum_{i=1}^{r} \hat{\mathbf{u}}'_i \hat{\mathbf{u}}'^H_i$$
$$\hat{\mathbf{\Pi}}'^{\perp}_c = \mathbf{I}_m - \hat{\mathbf{\Pi}}'_c \quad (16)$$

Finally, the adaptive filter $\hat{\mathbf{w}}'$ studied in this paper is:

$$\hat{\mathbf{w}}' = \hat{\mathbf{\Pi}}'^{\perp}_c \mathbf{d}' \quad (17)$$

We propose in next section to compute its theoretical performances.

3. THEORETICAL SNR LOSS

3.1. Definition of the SNR Loss

In the sequel, we assume that the target steering vector $\mathbf{d}'$ is normalized and does not belong to the clutter subspace:

$$\mathbf{S}'_{\Sigma}\mathbf{d}' = \mathbf{M}'\mathbf{d}' = \mathbf{\Pi}'_c\mathbf{d}' = \mathbf{0} \quad (18)$$

It follows from (10) and (12)

$$\begin{aligned} \mathbf{\Sigma}'\mathbf{d}' &= \lambda\mathbf{d}' \\ \mathbf{\Sigma}'^{-1}\mathbf{d}' &= \tfrac{1}{\lambda}\mathbf{d}' \\ \mathbf{\Pi}'^{\perp}_c\mathbf{d}' &= \mathbf{d}' \end{aligned} \quad (19)$$

Let us now compute the SNR Loss. The filter output is expressed as:

$$\mathbf{w}'^H\mathbf{x}' = \alpha\mathbf{w}'^H\mathbf{d}' + \mathbf{w}'^H\mathbf{n}' \quad (20)$$

The Signal to Noise Ratio, SNR_{out}, at the filter output is:

$$SNR_{out} = \frac{|\alpha|^2|\mathbf{w}'^H\mathbf{d}'|^2}{E\left[\mathbf{w}'^H\mathbf{n}'\mathbf{n}'^H\mathbf{w}'\right]} = \frac{|\alpha|^2|\mathbf{w}'^H\mathbf{d}'|^2}{\mathbf{w}'^H\mathbf{\Sigma}'\mathbf{w}'} \quad (21)$$

SNR_{out} is maximum when $\mathbf{w}' = \mathbf{w}'_{opt}$ and its value is:

$$SNR_{max} = |\alpha|^2\mathbf{d}'^H\mathbf{\Sigma}'^{-1}\mathbf{d}' \quad (22)$$

The SNR Loss, ρ, is the ratio between the SNR_{out}, computed for $\mathbf{w}' = \hat{\mathbf{w}}'$, and SNR_{max}. After some simplifications using (18) and (19), ρ is written as:

$$\rho = \frac{SNR_{out}}{SNR_{max}} = \lambda\frac{\left(\mathbf{d}'^H\hat{\mathbf{\Pi}}'^{\perp}_c\mathbf{d}'\right)^2}{\mathbf{d}'^H\hat{\mathbf{\Pi}}'^{\perp}_c\mathbf{\Sigma}'\hat{\mathbf{\Pi}}'^{\perp}_c\mathbf{d}'} \quad (23)$$

We propose in next section to evaluate the SNR Loss, ρ, using a perturbation analysis technique known to be valid for large K.

3.2. Perturbation Analysis

Actually, the estimation error $\Delta\mathbf{\Sigma}' = \hat{\mathbf{R}}' - \mathbf{\Sigma}'$ on $\mathbf{\Sigma}'$ induced a perturbation in the estimates $\hat{\mathbf{\Pi}}'_c$ and $\hat{\mathbf{\Pi}}'^{\perp}_c$ which is given up to the second order with respect to $\Delta\mathbf{\Sigma}'$ by:

$$\begin{aligned} \hat{\mathbf{\Pi}}'_c &= \mathbf{\Pi}'_c + \delta\mathbf{\Pi}'_c + \delta^2\mathbf{\Pi}'_c \\ \hat{\mathbf{\Pi}}'^{\perp}_c &= \mathbf{\Pi}'^{\perp}_c - \delta\mathbf{\Pi}'_c - \delta^2\mathbf{\Pi}'_c \end{aligned} \quad (24)$$

where $\delta\mathbf{\Pi}'_c$ and $\delta^2\mathbf{\Pi}'_c$ are defined as [11]:

$$\begin{aligned} \delta\mathbf{\Pi}'_c &= \mathbf{\Pi}'^{\perp}_c\Delta\mathbf{\Sigma}'\mathbf{M}' + \mathbf{M}'\Delta\mathbf{\Sigma}'\mathbf{\Pi}'^{\perp}_c \\ \delta^2\mathbf{\Pi}'_c &= \mathbf{\Pi}'^{\perp}_c\mathbf{\Gamma}\mathbf{M}' + \mathbf{M}'\mathbf{\Gamma}^*\mathbf{\Pi}'^{\perp}_c + \mathbf{\Pi}'_c\mathbf{\Phi}\mathbf{\Pi}'_c + \mathbf{\Pi}'^{\perp}_c\Delta\mathbf{\Sigma}'\mathbf{M}'^2\Delta\mathbf{\Sigma}'\mathbf{\Pi}'^{\perp}_c \end{aligned} \quad (25)$$

where matrices $\mathbf{\Gamma}$ and $\mathbf{\Phi}$ are second order terms in $\Delta\mathbf{\Sigma}'$. After some algebraic manipulations (not detailed due to a lack of space), we obtain the second order expression of ρ (23):

$$\begin{aligned} \rho &= 1 - \mathbf{d}'^H\hat{\mathbf{R}}'\left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)\hat{\mathbf{R}}'\mathbf{d}' \\ &= 1 - \|\left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2}\hat{\mathbf{R}}'\mathbf{d}'\|^2 \end{aligned} \quad (26)$$

To obtain a simpler expression of ρ, let us set:

$$\left(\frac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2} = \sum_{i=1}^{r} a_i\mathbf{u}'_i\mathbf{u}'^H_i \text{ with } a_i = \frac{1}{E\left[\tau\right]\lambda_i}\sqrt{\frac{E\left[\tau\right]\lambda_i + \lambda}{\lambda}} \quad (27)$$

and

$$\begin{aligned} \mathbf{z} &= \left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2}\hat{\mathbf{R}}'\mathbf{d}' \\ \mathbf{z}_k &= \mathcal{R}e\left(\left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2}\mathbf{x}'_k\mathbf{x}'^H_k\mathbf{d}'\right) \end{aligned} \quad (28)$$

Eq. (26) can be rewritten as:

$$\rho = 1 - \|\mathbf{z}\|^2 \text{ with } \mathbf{z} = \frac{1}{K'}\sum_{k=1}^{K'}\mathbf{z}_k, \quad (29)$$

with $K' = 2K$. For large K', as assumed in this paper, the Central Limit Theorem (CLT) ensures that $\mathbf{z}$ is Gaussian distributed. Its first and second order moments follow from those of $\mathbf{z}_k$ and will be now investigated. The SNR Loss distribution will be obtained from these results. The first order moment of $\mathbf{z}_k$ is:

$$\begin{aligned} E\left[\mathbf{z}_k\right] &= \mathcal{R}e\left(\left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2}E\left[\mathbf{x}'_k\mathbf{x}'^H_k\right]\mathbf{d}'\right) \\ &= \mathcal{R}e\left(\left(\tfrac{1}{\lambda}\mathbf{M}' + \mathbf{M}'^2\right)^{1/2}\mathbf{\Sigma}'\mathbf{d}'\right) = \mathbf{0}, \end{aligned} \quad (30)$$

since $\mathbf{u}'^H_i\mathbf{d}' = 0$ for $i \leq r$. Let us now derived the second order moments of $\mathbf{z}_k$. By setting:

$$\begin{aligned} &\mathbf{U} = [\mathbf{u}'_1 \dots \mathbf{u}'_r] \\ &\mathbf{y}_k = \mathbf{U}^H\mathbf{x}'_k \text{ and } (\mathbf{y}_k)_{r+1} = \mathbf{d}'^H\mathbf{x}'_k \;, \\ &\mathbf{y} = \mathbf{U}^H\mathbf{x}' \end{aligned} \quad (31)$$

each element of $\mathbf{y}_k$ can be written as follows:

$$\begin{aligned} (\mathbf{y}_k)_i &= \sqrt{(\tau_k\lambda_i + \lambda)\chi^1_{k,i}}\exp\left(j\theta_{k,i}\right) \quad i = 1, \dots, r \\ (\mathbf{y}_k)_{r+1} &= \sqrt{\lambda\chi^1_{k,r+1}}\exp\left(j\theta_{k,r+1}\right) \end{aligned} , \quad (32)$$

where $\chi^1_{k,i}$ is a Chi-square variable with 1 degree of freedom, $\theta_{k,i}$ is an uniform variable on $[0\,,2\pi]$, and all variables are independent. Therefore, we obtain:

$$\begin{aligned} \mathbf{z}_k &= \mathcal{R}e\left(\sum_{i=1}^{r} a_i(\mathbf{u}'^H_i\mathbf{x}'_k)(\mathbf{x}'^H_k\mathbf{d}')\mathbf{u}'_i\right) \\ &= \mathcal{R}e\left(\sum_{i=1}^{r} a_i(\mathbf{y}_k)_i(\mathbf{y}^*_k)_{r+1}\mathbf{u}'_i\right) \end{aligned} \quad (33)$$

The second order moments of $\mathbf{z}_k$ can be easily obtained from (32) and (33):

$$E\left[\mathbf{z}_k\mathbf{z}_k^H\right] = \sum_{i=1}^{r} a_i^2 E\left[(\tau_k\lambda_i + \lambda)\lambda\chi_i^1\chi_{r+1}^1\right]\mathbf{u}_i'\mathbf{u}_i'^H \quad (34)$$

where χ_i^1 and χ_{r+1}^1 are respectively two independent Chi-square variables with 1 degree of freedom.

The SNR Loss distribution is deduced from (29), (34) and the CLT. It can be shown that:

$$\rho = 1 - \frac{1}{K'}\sum_{i=1}^{r}\left(\frac{E\left[\tau\right]\lambda_i + \lambda}{E\left[\tau\right]\lambda_i}\right)^2\chi_i^1, \quad (35)$$

with $K' = 2K$. In the case of a strong clutter (a realistic hypothesis in STAP context), we have $E\left[\tau\right]\lambda_i \gg \lambda$ for $i = 1, \ldots, r$. Then, (35) can be reduced as:

$$E\left[\rho\right] \approx 1 - \frac{r}{2K} \quad (36)$$

By comparing this result to the classical result of [4, 5], we notice that a 3dB SNR Loss is reached for $K = r$, instead of $K = 2r$ when the persymmetric structure is not taken into account. Moreover, we notice that the final result does not depend of the texture τ. We illustrate this result in next section by means of two STAP simulations with simulated data and real data (containing real clutter and synthetic targets).

4. NUMERICAL SIMULATIONS

4.1. Validation of Theoretical SNR Loss

We consider the following STAP configuration to check the theoretical SNR Loss of Eq. (35). The number N of sensors is 8 and the number M of coherent pulses is also 8. The center frequency and the bandwidth are respectively equal to $f_0 = 450$ MHz and $B = 4$ MHz. The radar velocity is 100 m/s. The inter-element spacing is $d = \frac{c}{2f_0}$ (c is the celerity of light) and the pulse repetition frequency is $f_r = 600$ Hz. The clutter rank is computed from Reed-Mallet formula [1] and is equal to $r = 15$. Therefore, the clutter has a low-rank structure since $r = 15 < NM = 64$.

The CM of the Gaussian clutter, $\mathbf{C}$, is computed using the model presented in [1]. To simulate the SIRV clutter, we choose for the texture τ a Gamma distribution with shape parameter $\nu = 1$ and scale parameter $1/\nu = 1$ (which results in $E\left[\tau\right] = 1$). The identity matrix is next added to build the CM $\mathbf{\Sigma}$ of Eq. (6). The Clutter to Noise Ratio (CNR) is 25 dB. We next obtain $\mathbf{\Sigma}'$ of Eq. (10) by using the matrix $\mathbf{T}$. The eigendecomposition of this last matrix allows to obtain eigenvalues $\lambda_1, \ldots, \lambda_r, \lambda$ and therefore the theoretical SNR Loss of Eq. (35).

In the same STAP configuration, K secondary data have been simulated. These secondary data allow us to obtain the SCM $\hat{\mathbf{R}}_{SCM}$ and the persymmetric SCM $\hat{\mathbf{R}}'$ of Eq. (15). From its eigendecomposition, the sub-optimal STAP filter $\hat{\mathbf{w}}'$ of Eq. (17) has been computed and the SNR Loss of Eq. (23) has been evaluated using 1000 trials.

The same steps are used to evaluate the numerical and theoretical SNR Loss computed from the classical Low-Rank STAP filter built from the SCM. Theoretical result for Gaussian clutter is well known [5] and the result for SIRV clutter could be found in [14].

Figure 1 shows the numerical and the theoretical SNR Losses obtained from Low-Rank STAP filters built from SCM and persymmetric SCM for different values of K. We notice that the numerical SNR Losses are very close to the theoretical ones which validates the theoretical formula of Eq. (35). Moreover, we see that the Low-Rank STAP filter based on the persymmetric SCM yields better performance than the classical Low-Rank one.

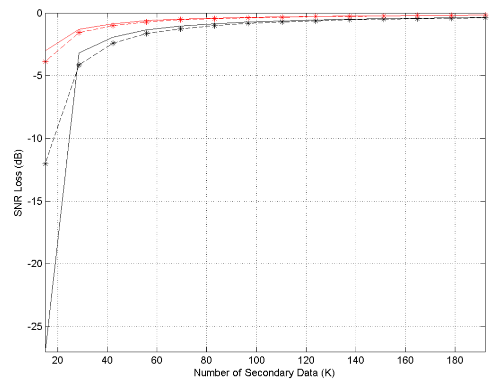

Fig. 1. Theoretical SNR Loss of LR STAP built from SCM (solid line black), Numerical SNR Loss of LR STAP built from SCM (dashed line * black), Theoretical SNR Loss of LR STAP built from Persymmetric SCM (solid line red), Numerical SNR Loss of LR STAP built from Persymmetric SCM (dashed line * red) in function of K.

4.2. Real Clutter Data

The STAP data are provided by the agency DGA/MI: the clutter is real but the targets are synthetic. The number of sensors is $N = 4$ and the number of coherent pulses is $M = 64$. The center frequency and the bandwidth are respectively equal to $f_0 = 10$ GHz and $B = 5$ MHz. The radar velocity is given by $V = 100$ m/s. The inter-element spacing is $d = 0.3$ m and the pulse repetition frequency is $f_r = 1$ kHz. For this particular STAP datacube, the clutter is fitted by our clutter data model of Eq. (3) since its statistic is shown slightly non-homogeneous [15]. The clutter to noise ratio is equal to 20 dB. In this scenario, a target with a signal to clutter ratio of -5 dB is present at (4 m/s, 0 deg, cell 216). The total number of secondary data is $K = 408$. The clutter rank obtained from Brennan's rule [13] is equal to $r = 45$. This value is small in comparison to the full size of clutter CM, $MN = 256$. The outputs of adaptive low-rank STAP filters, $\hat{\Lambda}_{LR-SCM} = |\mathbf{d}^H\hat{\mathbf{\Pi}}_c^{\perp}\mathbf{x}|^2$ and $\hat{\Lambda}'_{LR-SCM} = |\mathbf{d}'^H\hat{\mathbf{\Pi}}_c'^{\perp}\mathbf{x}'|^2$ (new LR STAP filter proposed in this paper), are used. Figures 2 and 3 show results of $\hat{\Lambda}_{LR-SCM}$ and $\hat{\Lambda}'_{LR-SCM}$ for respectively 100 (almost $2r$) et 50 (almost r) secondary data. As expected in the theoretical section, we notice that the persymmetry property allow to reduce the number of secondary data needed for a correct result.

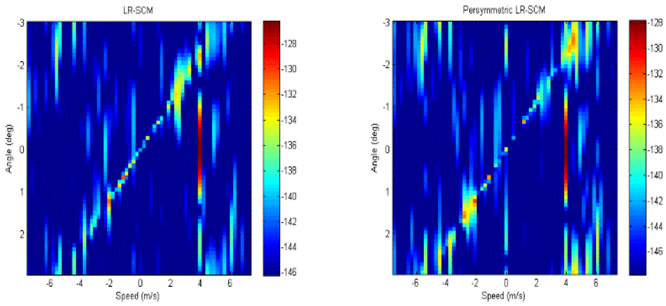

Fig. 2. $\hat{\Lambda}_{LR-SCM}$ (left) and $\hat{\Lambda}'_{LR-SCM}$ (right) with 100 cells to estimate the CM.

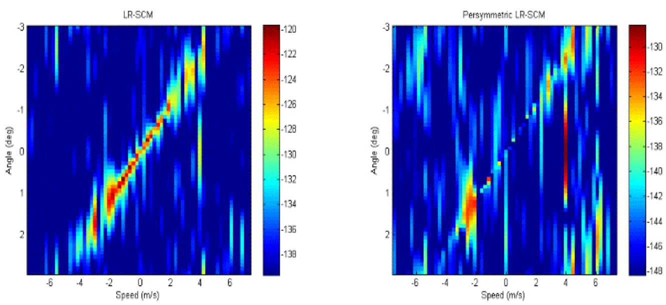

Fig. 3. $\hat{\Lambda}_{LR-SCM}$ (left) and $\hat{\Lambda}'_{LR-SCM}$ (right) with 50 cells to estimate the CM.

5. CONCLUSION

In this paper, we have proposed a new LR STAP filter by using the persymmetry property of the CM. This filter has been derived by using data transformed by a unitary matrix $\mathbf{T}$. Then it has been theoretically analyzed through the derivation of the SNR loss. Finally, in a context of a LR-SIRV clutter, the resulting STAP filter is shown, both theoretically and experimentally, to exhibit good performance with fewer secondary data: 3dB SNR loss is achieved with only r secondary data.

6. REFERENCES

[1] J. Ward, "Space-time adaptive processing for airborne radar," Tech. Rep., Lincoln Lab., MIT, Lexington, Mass., USA, December 1994.

[2] K. Yao, "A representation theorem and its applications to spherically invariant random processes," *IEE Trans. on Inf. Th.*, vol. 19, no. 5, pp. 600 – 608, September 1973.

[3] M. Rangaswamy, F.C. Lin, and K.R. Gerlach, "Robust adaptive signal processing methods for heterogeneous radar clutter scenarios," *Signal Processing*, vol. 84, pp. 1653 – 1665, 2004.

[4] I. Kirsteins and D. Tufts, "Adaptive detection using a low rank approximation to a data matrix," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 30, pp. 55 – 67, 1994.

[5] A. Haimovich, "Asymptotic distribution of the conditional signal-to-noise ratio in an eigenanalysis-based adaptive array," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 33, pp. 988 – 997, 1997.

[6] L. Cai and H. Wang, "A persymmetric multiband glr algorithm," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 30, no. 3, pp. 806–816, July 1992.

[7] E. Conte and A. De Maio, "Exploiting persymmetry for cfar detection in compound-gaussian clutter," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 39, no. 2, pp. 719 – 724, April 2003.

[8] E. Conte and A. De Maio, "Mitigation techniques for non-gaussian sea clutter," *IEEE Journal of Oceanic Engineering*, vol. 29, no. 2, pp. 284 – 302, 2004.

[9] G. Pailloux, P. Forster, J.P. Ovarlez, and F. Pascal, "On persymmetric covariance matrices in adaptive detection," in *Proceedings of ICASSP*, Las Vegas, Nevada, USA, April 2008.

[10] G. Pailloux, P. Forster, J.P. Ovarlez, and F. Pascal, "Persymmetric adaptive radar detectors," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 47, no. 4, pp. 2376–2390, October 2011.

[11] H. Krim, P. Forster, and J.G. Proakis, "Operator approach to performance analysis of root-music and root-min-norm," *IEEE Trans. on Sig. Proc.*, vol. 40, no. 7, pp. 1687 – 1696, July 1992.

[12] M. Rangaswamy, D.D. Weiner, and A. Ozturk, "Non-gaussian vector identification using spherically invariant random processes," *IEEE Trans.-AES*, vol. 29, no. 1, pp. 111–124, January 1993.

[13] L. E. Brennan and F.M. Staudaher, "Subclutter visibility demonstration," Tech. Rep., RL-TR-92-21, Adaptive Sensors Incorporated, March 1992.

[14] G. Ginolhac, P. Forster, F. Pascal, and J.P. Ovarlez, "Performance and robustness of two low-rank stap filters in a heterogeneous noise," *Submitted in IEEE Trans. on Sig. Proc.*, 2012.

[15] S. Marcos and S. Beau, "Stap à rang réduit, récursif en distance et utilisant un développement de taylor," *Traitement du Signal*, vol. 28, no. 1-2, pp. 171–201, 2011.