ROBUST MIMO RADAR DETECTION FOR CORRELATED SUBARRAYS

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ABSTRACT

Previously, the well-known Optimum Gaussian Detector (OGD) has been extended to the Multiple-Input Multiple-Output (MIMO) case where all transmit-receive subarrays are considered jointly as a system such that only one detection threshold is used [1, 2]. In this extension, all subarrays have been assumed to be widely separated and the transmitted waveforms are assumed to be orthogonal. However, the necessary separation needed for each subarray to be uncorrelated depends on several factors and it might not be possible to ensure that this condition is always respected, especially in the case of moving platforms. Moreover, perfectly orthogonal waveforms do not exist. Hence, we consider in this paper, a new robust MIMO detector that is able to maintain the same Probability of False Alarm irregardless of the correlation between the subarrays.

***Index Terms*—** Robust, MIMO Radar, Probability of False Alarm, Correlation

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) is a technique used in communications that has recently been adopted for radar applications [3]. In the context of radar, a (statistical) MIMO radar is one where both the transmit and receive elements are sufficiently separated so as to provide spatial diversity. This reduces the fluctuations of the target Radar Cross Section (RCS) due to the different target aspects seen by each pair of transmit-receive elements [4]. It can also be used to improve the probability of detection [5] and resolutions [6]. On top of that, each transmit element sends a different (orthogonal) waveform which can be separated at the receive end. This provides waveform diversity which in turn increases the separation between clutter and target returns [7].

In [8], we have considered a signal model where all the subarrays are assumed to be uncorrelated. Under Gaussian clutter, the optimum detector is the MIMO Optimum Gaussian Detector (MIMO OGD) [1, 2]. However, it is not always possible to have subarrays that are totally uncorrelated.

According to [1], the subarrays have to be sufficiently spaced in order to decorrelate the signal returns in each subarray. This might not be possible to respect this condition, especially when we consider MIMO-STAP where the transmit and/or receive subarrays are moving. Moreover, perfectly orthogonal waveforms do not exist, especially in the presence of Doppler frequency. In this paper, we assume that insufficient spacing between subarrays and imperfect orthogonality of the transmitted waveforms introduce correlation between the subarrays.

The aim of this paper is to derive a robust detector that is able to obtain the optimal result whether the subarrays are uncorrelated or not. Here we consider the case where the clutter is Gaussian. Using the Maximum Likelihood (ML) method, we obtain a new Gaussian MIMO detector which is robust to the correlation between the subarrays. It is found that this new detector has the same statistical properties in the absence of target irregardless if the subarrays are correlated or not. It becomes the MIMO OGD when the subarrays are uncorrelated.

This paper is organized as follows. Firstly, we present the new signal model for MIMO radars where subarrays can be correlated or not (Section 2). Using this signal model, we derive a new robust detector in Section 3. The adaptive version of this new detector is then derived based on Kelly's Test in Section 4. The Probability of False Alarm (P_{fa}) for this new detector is then analyzed through Monte-Carlo simulations for the cases where the clutter is correlated and not (Section 5). Finally, conclusions are presented in Section 6.

2. NEW SIGNAL MODEL

In [8], the transmit and receive subarrays are assumed to be widely separated. Orthogonal waveforms are transmitted such that the received signal after matched filtering can be expressed as:

$$\mathbf{y}_i = \alpha_i \mathbf{p}_i + \mathbf{z}_i \qquad \forall\, i = 1, \ldots, K$$

where K is the number of subarrays, α_i is the RCS of the target seen by the i-th subarray. $\mathbf{p}_i$ is the L_ix1 steering vector for the i-th subarray and L_i is the number of antenna elements.

However, according to [1], the required spacing between the subarrays for the target returns to be uncorrelated depends on the size of the target. Similarly, for the clutter returns to be uncorrelated, it depends on the resolution cell size. It is not always possible to achieve these spacings, especially in the case

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of MIMO-STAP where the transmit or receive subarrays are moving. More importantly, perfectly orthogonal waveforms do not exist. We assume that these factors cause the return signals from different subarrays to be correlated. Thus, we consider the following signal model that does not assume the independence of the different subarrays.

$$\mathbf{y} = \mathbf{s} + \mathbf{z},$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} \qquad \mathbf{s} = \begin{bmatrix} \alpha_1 \mathbf{p}_1 \\ \vdots \\ \alpha_K \mathbf{p}_K \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_K \end{bmatrix}.$$

The vectors $\mathbf{y}$, $\mathbf{p}$ and $\mathbf{z}$ are the concatenation of all the received signals, steering vectors and clutter returns, respectively. The covariance matrix of each $\mathbf{z}_i$ is given by $\mathbf{M}_{ii}$ while the inter-correlation matrix between $\mathbf{z}_i$ and $\mathbf{z}_j$ is denoted as $\mathbf{M}_{ij}$ such that $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{M})$ and $\sim$ means to be distributed as.

3. NEW ROBUST MIMO DETECTOR

Under Gaussian assumptions, we have the following hypothesis test:

$$\begin{cases} \mathrm{H}_0: & \mathbf{y} = \mathbf{z}, \\ \mathrm{H}_1: & \mathbf{y} = \mathbf{s} + \mathbf{z}. \end{cases}$$

Under the hypothesis H_0, it is assumed that the received signal contains only clutter returns and hence there is no target. Under the hypothesis H_1, it is assumed that the received signal contains a deterministic signal on top of the clutter returns and hence a target is present.

The classical likelihood ratio test is given by:

$$\Lambda(\mathbf{y}) = \frac{p(\mathbf{y}|\mathrm{H}_1)}{p(\mathbf{y}|\mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \eta.$$

Given that $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{M})$ and after some simple manipulations, the GLRT becomes:

$$\ln \Lambda(\mathbf{y}) = -(\mathbf{y} - \mathbf{s})^\dagger \mathbf{M}^{-1} (\mathbf{y} - \mathbf{s}) + \mathbf{y}^\dagger \mathbf{M}^{-1} \mathbf{y}$$

where the superscript $\dagger$ denotes the Hermitian operator. As the target amplitudes are unknown, we have to find their ML estimates: $\hat{\boldsymbol{\alpha}} = [\hat{\alpha_1} \ldots \hat{\alpha_1}]^T$ where the superscript T stands for transpose. For each α_i,

$$\begin{aligned} \frac{d \ln \Lambda(\mathbf{y})}{d\alpha_i} &= \mathrm{tr}\left[\left(\frac{d \ln \Lambda(\mathbf{y})}{d\mathbf{s}}\right)^\dagger \frac{d\mathbf{s}}{d\alpha_i}\right], \\ &= \left[2\mathbf{M}^{-1}(\mathbf{y} - \mathbf{s})\right]^\dagger \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{p}_i \\ \vdots \\ \mathbf{0} \end{bmatrix} = 0 \end{aligned}$$

where $\mathrm{tr}(\cdot)$ denotes the trace. This gives:

$$\mathbf{y}^\dagger \mathbf{M}^{-1} \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{p}_i \\ \vdots \\ \mathbf{0} \end{bmatrix} = \mathbf{s}^\dagger \mathbf{M}^{-1} \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{p}_i \\ \vdots \\ \mathbf{0} \end{bmatrix}. \tag{1}$$

Let $\mathbf{R} = \mathbf{M}^{-1}$ and $\mathbf{R}_{ij}$ is the block matrix corresponding to the correlation between subarrays i and j. Using $\mathbf{R}$, we can express Eqn. (1) as summations:

$$\begin{aligned} \sum_{j=1}^{K} \mathbf{y}_j^\dagger \mathbf{R}_{ji} \mathbf{p}_i &= \sum_{j=1}^{K} \mathbf{s}_j^\dagger \mathbf{R}_{ji} \mathbf{p}_i = \sum_{j=1}^{K} \hat{\alpha}_j^* \mathbf{p}_j^\dagger \mathbf{R}_{ji} \mathbf{p}_i, \\ \sum_{j=1}^{K} \mathbf{p}_i^\dagger \mathbf{R}_{ij} \mathbf{y}_j &= \sum_{j=1}^{K} \hat{\alpha}_j \mathbf{p}_i^\dagger \mathbf{R}_{ij} \mathbf{p}_j \qquad \forall\, i = 1 \ldots K. \end{aligned}$$

Re-expressing this in matrix form, we get:

$$\mathbf{A}\hat{\boldsymbol{\alpha}} = \mathbf{b}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{p}_1^\dagger \mathbf{R}_{11} \mathbf{p}_1 & \ldots & \mathbf{p}_1^\dagger \mathbf{R}_{1K} \mathbf{p}_K \\ \vdots & \ddots & \vdots \\ \mathbf{p}_K^\dagger \mathbf{R}_{K1} \mathbf{p}_1 & \ldots & \mathbf{p}_K^\dagger \mathbf{R}_{KK} \mathbf{p}_K \end{bmatrix}, \tag{2}$$

$$\mathbf{b} = \begin{bmatrix} \sum_{i=1}^{K} \mathbf{p}_1^\dagger \mathbf{R}_{1i} \mathbf{y}_i \\ \vdots \\ \sum_{i=1}^{K} \mathbf{p}_K^\dagger \mathbf{R}_{Ki} \mathbf{y}_i \end{bmatrix}.$$

Note that $\mathbf{A}$ is Hermitian and positive definite. Hence the ML estimate of $\boldsymbol{\alpha}$ is given by:

$$\hat{\boldsymbol{\alpha}} = \mathbf{A}^{-1}\mathbf{b}. \tag{3}$$

Using these new notations as well as Eqn. (3), the detector can be expressed as:

$$\begin{aligned} \ln \Lambda(\mathbf{y}) &= 2\Re(\mathbf{y}^\dagger \mathbf{M}^{-1} \mathbf{s}) - \mathbf{s}^\dagger \mathbf{M}^{-1} \mathbf{s}, \\ &= 2\Re(\mathbf{b}^\dagger \boldsymbol{\alpha}) - \hat{\boldsymbol{\alpha}}^\dagger \mathbf{A} \hat{\boldsymbol{\alpha}}, \\ &= \mathbf{b}^\dagger \mathbf{A}^{-1} \mathbf{b} \end{aligned}$$

where $\Re(\cdot)$ denotes the real part. We can also express $\mathbf{b}$ as $\mathbf{b} = \mathbf{P}^\dagger \mathbf{M}^{-1} \mathbf{y}$ where $\mathbf{P}$ is a $(\sum_{i=1}^{K} L_i)$xK matrix containing all the steering vectors:

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \ldots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{p}_K \end{bmatrix}.$$

Thus the detector becomes:

$$\ln \Lambda(\mathbf{y}) = \mathbf{y}^\dagger \mathbf{M}^{-1} \mathbf{P} \mathbf{A}^{-1} \mathbf{P}^\dagger \mathbf{M}^{-1} \mathbf{y}.$$

This detector will be referred to as the Robust MIMO OGD (R-MIMO OGD) detector.

Remark 3.1 *In the particular case where there is no correlation between the subarrays, the R-MIMO OGD detector becomes:*

$$\ln\Lambda(\mathbf{y}) = \sum_{i=1}^{K} \frac{|\mathbf{p}_i^\dagger \mathbf{M}_{ii}^{-1}\mathbf{y}_i|^2}{\mathbf{p}_i^\dagger \mathbf{M}_{ii}^{-1}\mathbf{p}_i}. \quad (4)$$

This is the same equation as that obtained for MIMO OGD, as proposed in [1, 2].

Indeed, when the subarrays are uncorrelated, $\mathbf{M}$ is block diagonal:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{M}_{KK} \end{bmatrix}.$$

Its inverse is also block diagonal. $\mathbf{b}$ and $\mathbf{A}$ become:

$$\begin{aligned} \mathbf{b} &= \begin{bmatrix} \mathbf{p}_1^\dagger \mathbf{M}_{11}^{-1}\mathbf{y}_1 \\ \vdots \\ \mathbf{p}_K^\dagger \mathbf{M}_{KK}^{-1}\mathbf{y}_K \end{bmatrix}, \\ \mathbf{A} &= \begin{bmatrix} \mathbf{p}_1^\dagger \mathbf{M}_{11}^{-1}\mathbf{p}_1 & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{p}_K^\dagger \mathbf{M}_{KK}^{-1}\mathbf{p}_K \end{bmatrix}. \end{aligned}$$

Thus the R-MIMO OGD detector becomes like that in Eqn. (4).

Under H_0, the received signal contains only clutter. Consider the whitened received signal $\mathbf{x} = \mathbf{M}^{-1/2}\mathbf{y}$, the detector becomes:

$$\ln\Lambda(\mathbf{y}) = \mathbf{x}^\dagger \mathbf{M}^{-1/2}\mathbf{P}\mathbf{A}^{-1}\mathbf{P}^\dagger \mathbf{M}^{-1/2}\mathbf{x}.$$

As the matrices $\mathbf{M}$, $\mathbf{A}$ and $\mathbf{P}$ are of full rank and the minimum rank among these matrices is K, the rank of the matrix $\mathbf{M}^{-1/2}\mathbf{P}\mathbf{A}^{-1}\mathbf{P}^\dagger\mathbf{M}^{-1/2}$ is K. As $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, the distribution of the detector under H_0 is simply χ^2_{2K} which is the same as the distribution of the MIMO OGD detector where the subarrays are not correlated. More importantly, the distribution does not depend on the correlation between the subarrays. This shows the $\mathbf{M}$-Constant False Alarm Rate ($\mathbf{M}$-CFAR) property of the R-MIMO OGD detector. This property is very useful as it means that the requirement of independence between subarrays can be relaxed using this detector.

4. ADAPTIVE MIMO DETECTOR

As the covariance matrix is usually unknown in reality, we consider in this section the adaptive version of the detector.

Theorem 4.1 *Given a MIMO radar system containing K sub-systems and L_i elements ($L > 1$) in each sub-system, the optimum adaptive detector using N_r secondary data (containing only clutter returns) is:*

$$\frac{\mathbf{y}^\dagger \hat{\mathbf{M}}^{-1}\mathbf{P}\hat{\mathbf{A}}^{-1}\mathbf{P}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y}}{N_r + \mathbf{y}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_2. \quad (5)$$

Remark 4.1

- *If there is only one subarray, Eqn. (5) becomes Kelly's Test [9].*
- *If the covariance matrix $\mathbf{M}$ is block diagonal, i.e. the subarrays are uncorrelated, and this* a priori *information is used in the estimation of $\mathbf{M}$, this will result in another version of the MIMO Kelly' s Test:*

$$\prod_{i=1}^{K} \frac{1}{1 - \frac{|\mathbf{p}_i^\dagger\hat{\mathbf{M}}_{ii}^{-1}\mathbf{y}_i|^2}{(\mathbf{p}_i^\dagger\hat{\mathbf{M}}_{ii}^{-1}\mathbf{p}_i)(N_r + \mathbf{y}_i^\dagger\hat{\mathbf{M}}_{ii}^{-1}\mathbf{y}_i)}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_3.$$

Proof 4.1 *Taking into consideration both the received signal and the secondary data, the likelihood ratio test can be written as:*

$$\max_{\boldsymbol{\alpha}} \Lambda(\mathbf{y}) = \frac{\|\mathbf{T}_0\|}{\min_{\boldsymbol{\alpha}}\|\mathbf{T}_1\|} \quad (6)$$

where $\|\cdot\|$ denotes the determinant,

$$\begin{aligned} \mathbf{T}_0 &= \frac{1}{N_r+1}\left(\mathbf{y}\mathbf{y}^\dagger + \sum_{l=1}^{Nr}\mathbf{c}(l)\mathbf{c}(l)^\dagger\right), \\ \mathbf{T}_1 &= \frac{1}{N_r+1}\left((\mathbf{y}-\mathbf{s})(\mathbf{y}-\mathbf{s})^\dagger + \sum_{l=1}^{Nr}\mathbf{c}(l)\mathbf{c}(l)^\dagger\right) \end{aligned}$$

and $\mathbf{c}(l)$ are the secondary data containing only clutter returns.

Replacing $\|\mathbf{T}_0\|$ and $\|\mathbf{T}_1\|$ in Eqn. (6), we get:

$$\begin{aligned} \max_{\boldsymbol{\alpha}} \Lambda(\mathbf{y}) &= \frac{1 + \frac{1}{N_r}\mathbf{y}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y}}{\min_{\boldsymbol{\alpha}}(1 + \frac{1}{N_r}(\mathbf{y}-\mathbf{s})^\dagger\hat{\mathbf{M}}^{-1}(\mathbf{y}-\mathbf{s}))}, \\ &= \frac{1 + \frac{1}{N_r}\mathbf{y}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y}}{1 + \frac{1}{N_r}\mathbf{y}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y} - \frac{1}{N_r}\mathbf{y}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{P}\hat{\mathbf{A}}^{-1}\mathbf{P}^\dagger\hat{\mathbf{M}}^{-1}\mathbf{y}} \end{aligned}$$

where $\hat{\mathbf{M}}$ is the Sample Covariance Matrix of $\mathbf{M}$ and is given by:

$$\hat{\mathbf{M}} = \frac{1}{N_r}\sum_{l=1}^{Nr}\mathbf{c}(l)\mathbf{c}(l)^\dagger$$

and $\hat{\mathbf{A}}$ is the matrix obtained by replacing $\mathbf{M}$ by $\hat{\mathbf{M}}$ in Eqn. (2). Re-arranging the equation, we obtain the MIMO Kelly's Test given in Eqn. (5).

5. SIMULATION RESULTS

Monte-Carlo simulations (10^6 runs) are carried out. The number of subarrays K is set to be 3 and the number of elements in each subarray L_i is set to be identical and equal to $L = 6$. The covariance matrix $\mathbf{M}_{ii}$ of each $\mathbf{z}_i$, without loss of generalities, is chosen identically and equal to $\mathbf{M}_{sa}$. $\mathbf{M}_{sa}$ is spatially colored and its elements are given by:

$$\mathrm{M}_{sa}(p,q) = \rho^{|p-q|}e^{j\frac{\pi}{2}(p-q)}.$$

The correlation coefficient ρ is chosen to be 0.2 such that there is a slight correlation between different elements of the subarray.

For the case where the subarrays are considered to be uncorrelated, we have

$$\mathbf{M}_{ij} = \begin{cases} \mathbf{M}_{sa} & i = j, \\ \mathbf{0} & i \neq j. \end{cases}$$

For the case where the subarrays are correlated, the intercorrelation matrices are generated using uniformly distributed variables:

$$\mathrm{M}_{ij}(p,q) = \tau_{ij}\rho_{ij}^{|p-q|}e^{j\frac{\pi}{2}(p-q)}.$$

where τ_{ij} and ρ_{ij} are uniformly distributed in the interval $[0, 0.4]$. Thus, the mean of ρ_{ij} is equal to ρ. The inclusion of τ_{ij} is to make the power of the intercorrelation matrices M_{ij} smaller than that of the correlation matrices M_{ii}.

In Fig. 1, we have plotted the "P_{fa}-λ" curves for the R-MIMO OGD detector when the subarrays are correlated and when they are uncorrelated. We see that there is perfect agreement between the two curves showing that the curves do not depend on the correlation between the subarrays, i.e. it does not depend on the covariance matrix. This confirms the M-CFAR property of the new R-MIMO OGD detector.

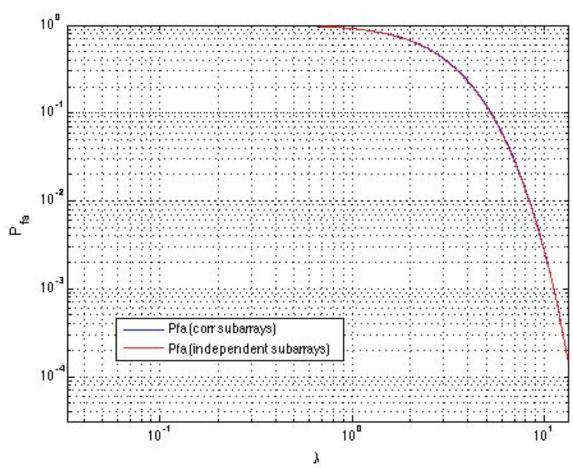

Fig. 1. P_{fa} against detection threshold λ for Monte-Carlo simulations under Gaussian clutter under R-MIMO OGD for correlated and uncorrelated subarrays.

6. CONCLUSIONS

A new MIMO Gaussian detector which takes into consideration possible correlation between subarrays has been derived. It becomes the classical MIMO OGD when the subarrays are uncorrelated such that the covariance matrix is block diagonal. This new detector is robust as its statistical property under H_0 is the same no matter if there is correlation between subarrays or not, i.e. the detector is M-CFAR. The adaptive version of this detector is also derived based on Kelly's Test. Finally, the M-CFAR property of the detector is verified using Monte-Carlo simulations.

This new detector is interesting due to its robustness. Further works will be done to look into the statistical properties and detection performance of the detector and its adaptive version.

7. REFERENCES

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