HIERARCHICAL SEGMENTATION OF POLARIMETRIC SAR IMAGES USING HETEROGENEOUS CLUTTER MODELS

L. Bombrun[1], *J.-M. Beaulieu*[2], *G. Vasile*[1], *J.-P. Ovarlez*[3,4], *F. Pascal*[4] *and M. Gay*[1]

[1] : Grenoble-Image-sPeach-Signal-Automatics Lab, CNRS / Grenoble-INP, FRANCE,
{*lionel.bombrun*|*gabriel.vasile*|*michel.gay*}*@gipsa-lab.grenoble-inp.fr*

[2] : Computer Science and Software Engineering Department, Laval University, CANADA, *jean-marie.beaulieu@ift.ulaval.ca*

[3] : French Aerospace Lab, ONERA DEMR/TSI, FRANCE, *ovarlez@onera.fr*

[4] : SONDRA, Supélec, FRANCE, *frederic.pascal@supelec.fr*

ABSTRACT

In this paper, heterogeneous clutter models are introduced to describe Polarimetric Synthetic Aperture Radar (PolSAR) data. Based on the Spherically Invariant Random Vectors (SIRV) estimation scheme, the scalar texture parameter and the normalized covariance matrix are extracted. If the texture parameter is modeled by a Fisher PDF, the observed target scattering vector follows a KummerU PDF. Then, this PDF is implemented in a hierarchical segmentation algorithm. Segmentation results are shown on high resolution PolSAR data at L and X band.

***Index Terms*—** Fisher PDF, KummerU PDF, PolSAR data, Segmentation, Spherically Invariant Random Vectors.

1. INTRODUCTION

PolSAR images are generally modeled by a zero mean multivariate circular Gaussian distribution. Landcover backscatter characteristics are assumed homogeneous over the target area. However, thinner spatial features can be observed from the high resolution of newly available spaceborne and airborne SAR images. In this case, heterogeneous clutter models should be used because each resolution cell contains only a small number of scatterers.

In this paper, we propose to apply the SIRV estimation scheme in a hierarchical segmentation algorithm. This algorithm is based on the maximization of the SIRV log-likelihood function.

Here, we propose to use the Fisher Probability Density Function (PDF) to model the estimated texture parameter. For a Fisher distributed texture, we prove that the target scattering vector $\mathbf{k}$ follows a KummerU PDF and we implement this PDF in a hierarchical segmentation algorithm. Segmentation results are shown on high resolution PolSAR data over the Oberpfaffenhofen (L-band, ESAR) and Toulouse (X-band, RAMSES) test-sites.

2. SIRV MODEL

2.1. Definition

With the new generation of airborne and spaceborne SAR sensors, the number of scatterers present in each resolution cell decreases considerably, homogeneous hypothesis of the PolSAR clutter can be reconsidered. Heterogeneous clutter models have therefore recently been studied.

In 1973, Kung Yao has first introduced the use of SIRV and their applications to estimation and detection in communication [1]. From a PolSAR point of view, the target vector $\mathbf{k}$ is defined as the product of a square root of a positive random variable τ (representing the texture) with an independent complex Gaussian vector $\mathbf{z}$ with zero mean and covariance matrix $[M] = E\{\mathbf{z}\mathbf{z}^H\}$ (representing the speckle) :

$$\mathbf{k} = \sqrt{\tau}\,\mathbf{z} \tag{1}$$

where the superscript H denotes the complex conjugate transposition and $E\{\cdot\}$ the mathematical expectation.

For a given covariance matrix $[M]$, the ML estimator of the texture parameter τ is given by :

$$\hat{\tau}_i = \frac{\mathbf{k}_i^H [M]^{-1} \mathbf{k}_i}{p}. \tag{2}$$

where p is the dimension of the target scattering vector $\mathbf{k}$ ($p = 3$ for the reciprocal case).

The ML estimator of the normalized covariance matrix under the deterministic texture case is the solution of the following recursive equation :

$$[\hat{M}]_{FP} = f([\hat{M}]_{FP}) = \frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^H}{\mathbf{k}_i^H [\hat{M}]_{FP}^{-1} \mathbf{k}_i}. \tag{3}$$

Pascal et al. have established the existence and the uniqueness, up to a scalar factor, of the Fixed Point estimator of the normalized covariance matrix, as well as the convergence of the recursive algorithm whatever the initialization [2] [3]. In this paper, the trace of the covariance matrix is normalized to p the dimension of target scattering vector.

It is important to notice that in the SIRV definition, the PDF of the texture random variable is not explicitly specified. As a consequence, SIRVs describe a whole class of stochastic processes. This class includes the conventional clutter models having Gaussian, $\mathcal{K}$-distributed, Rayleigh or Weibull PDFs.

2.2. Texture modeling

2.2.1. Definition

The texture parameter τ is the random power of the clutter, it characterizes the randomness induced by variations in the radar backscattering over different polarization channels. This texture parameter can be rewritten as the product of a normalized texture parameter ξ with the mean backscattered power μ by :

$$\tau = \mu\,\xi \tag{4}$$

For a segment S containing N pixels, the normalized texture parameter for pixel i is defined by :

$$\xi_i = \frac{\tau_i}{\mu} = \frac{\tau_i}{\dfrac{1}{N}\sum_{j=1}^{N}\tau_j} = \frac{\mathbf{k}_i^H[M]^{-1}\mathbf{k}_i}{\dfrac{1}{N}\sum_{j=1}^{N}\mathbf{k}_j^H[M]^{-1}\mathbf{k}_j} \tag{5}$$

2.2.2. Beta Prime PDF

Let ξ be a positive random variable distributed according to a Beta Prime distribution. Its PDF is defined by two parameters $\mathcal{L}$ and $\mathcal{M}$ as :

$$\mathcal{BP}\left[\xi|\mathcal{L},\mathcal{M}\right] = \frac{\Gamma(\mathcal{L}+\mathcal{M})}{\Gamma(\mathcal{L})\Gamma(\mathcal{M})}\frac{\xi^{\mathcal{L}-1}}{(1+\xi)^{\mathcal{L}+\mathcal{M}}} \tag{6}$$

If ξ follows a Beta Prime PDF with $\mathcal{L}$ and $\mathcal{M}$ parameters, the texture parameter τ is Fisher distributed with $m = \dfrac{\mu\mathcal{L}}{\mathcal{M}}$, $\mathcal{L}$ and $\mathcal{M}$ parameters.

2.2.3. Fisher PDF

The Fisher PDF is the Pearson type VI distribution, it is defined by three parameters as the Mellin convolution of a Gamma PDF by an Inverse Gamma PDF by [4] :

$$\begin{aligned}\mathcal{F}\left[\tau|m,\mathcal{L},\mathcal{M}\right] &= \mathcal{G}\left[m,\mathcal{L}\right]\,\hat{\star}\,\mathcal{GI}\left[1,\mathcal{M}\right]\\ &= \frac{\Gamma(\mathcal{L}+\mathcal{M})}{\Gamma(\mathcal{L})\Gamma(\mathcal{M})}\frac{\mathcal{L}}{\mathcal{M}m}\frac{\left(\dfrac{\mathcal{L}\tau}{\mathcal{M}m}\right)^{\mathcal{L}-1}}{\left(1+\dfrac{\mathcal{L}\tau}{\mathcal{M}m}\right)^{\mathcal{L}+\mathcal{M}}}\end{aligned} \tag{7}$$

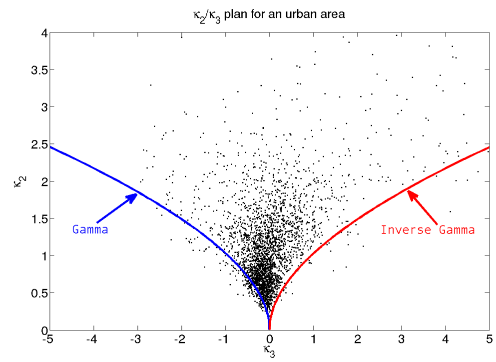


Fig. 1. κ_2/κ_3 plan for an urban area over the Oberpfaffenhofen test-site (ESAR, L-band).

2.2.4. Parameter estimation

As the Fisher PDF is linked with second kind statistics, recent works have proposed to estimate Fisher parameters with the log-cumulants method. Nevertheless, they are not ML estimators. By working on the normalized texture parameter ξ, ML Beta Prime estimators can be computed numerically. Then, according to relation shown in section 2.2.2, ML Fisher estimators are obtained.

2.2.5. Benefit of Fisher PDF

An urban area (80×35 pixels) from the L-band ESAR data over the Oberpfaffenhofen test-site has been extracted. Then, the covariance matrix $[M]_{FP}$ and the texture parameter τ are estimated according to Eq.2 and Eq. 3. To see the benefit of Fisher PDF to model PolSAR clutter, the κ_2/κ_3 plan has been plotted on Fig. 1. It shows the evolution of the second log-cumulant versus the third log-cumulant. In this plan, Gamma and Inverse Gamma PDF are respectively represented by the blue and red line. Fisher PDF cover all the space between the blue and red line.

This example shows that Fisher PDF are well adapted to model PolSAR clutter.

2.3. Target scattering PDF for a Fisher distributed clutter

For a Fisher distributed clutter, one can prove that the target scattering vector PDF can be expressed with the density generator function $h_p\left(\mathbf{k}^H[M]^{-1}\mathbf{k}\right)$ by [5] :

$$p_{\mathbf{k}}(\mathbf{k}|[M],\mathcal{L},\mathcal{M},m) = \frac{1}{\pi^p|[M]|}h_p\left(\mathbf{k}^H[M]^{-1}\mathbf{k}\right) \tag{8}$$

where the density generator function $h_p(\cdot)$ is defined by [6] :

$$h_p\left(\mathbf{k}^H[M]^{-1}\mathbf{k}\right) = \frac{\Gamma(\mathcal{L}+\mathcal{M})}{\Gamma(\mathcal{L})\Gamma(\mathcal{M})}\left(\frac{\mathcal{L}}{\mathcal{M}m}\right)^p\Gamma(p+\mathcal{M})U\left(a;b;z\right) \tag{9}$$

with $a = p + \mathcal{M}$, $b = 1 + p - \mathcal{L}$ and $z = \dfrac{\mathcal{L}}{\mathcal{M}m}\mathbf{k}^H[M]^{-1}\mathbf{k}$.

$|\cdot|$ and $U(\cdot;\cdot;\cdot)$ denotes respectively the determinant operator and the confluent hypergeometric function of the second kind (KummerU).

In the following, this PDF is named the KummerU PDF.

2.4. Maximum Likelihood (ML) estimator

The exact ML estimator of the normalized covariance matrix depends on the texture PDF, its expression is linked with the density generator function by :

$$[\hat{M}_{ML}] = \frac{1}{N}\sum_{i=1}^{N} \frac{h_{p+1}\left(\mathbf{k}_i^H[\hat{M}_{ML}]^{-1}\mathbf{k}_i\right)}{h_p\left(\mathbf{k}_i^H[\hat{M}_{ML}]^{-1}\mathbf{k}_i\right)}\,\mathbf{k}_i\mathbf{k}_i^H \qquad (10)$$

Chitour and Pascal have proved that Eq. 10 admits a unique solution and that its corresponding iterative algorithm converges to the Fixed Point solution for every admissible initial condition [7].

For a Fisher distributed clutter, one can replace the density generator function by its expression given in Eq. 9. It yields :

$$\begin{aligned}[\hat{M}_{ML}] &= \frac{p+\mathcal{M}}{N}\left(\frac{\mathcal{L}}{\mathcal{M}m}\right) \\ &\times\sum_{i=1}^{N} \frac{U\left(p+1+\mathcal{M}; 2+p-\mathcal{L}; \frac{\mathcal{L}}{\mathcal{M}m}\mathbf{k}_i^H[\hat{M}_{ML}]^{-1}\mathbf{k}_i\right)}{U\left(p+\mathcal{M}; 1+p-\mathcal{L}; \frac{\mathcal{L}}{\mathcal{M}m}\mathbf{k}_i^H[\hat{M}_{ML}]^{-1}\mathbf{k}_i\right)}\,\mathbf{k}_i\mathbf{k}_i^H\end{aligned} \qquad (11)$$

3. HIERARCHICAL SEGMENTATION

3.1. Principle

In this paper, the hierarchical segmentation algorithm proposed by Beaulieu and Touzi [8] is adapted to the KummerU distributed target scattering vector. The segmentation process can be divided into three steps :

1. Definition of an initial partition.
2. For each 4-connex segments pair, the Stepwise Criterion (SC) is computed. Then, the two segments which minimize the criterion are found and merged.
3. Stop if the maximum number of merges is reached, otherwise go to step 2.

3.2. Stepwise Criterion

The criterion used in the hierarchical algorithm is based on the log-likelihood function. The hierarchical segmentation algorithm merges the two adjacent segments S_i and S_j which minimizes the loss of likelihood. The stepwise criterion ($\mathrm{SC}_{i,j}$) can be expressed as [8]:

$$\mathrm{SC}_{i,j} = \mathrm{MLL}(S_i) + \mathrm{MLL}(S_j) - \mathrm{MLL}(S_i \cup S_j) \qquad (12)$$

where $\mathrm{MLL}(\cdot)$ denotes the segment maximum log-likelihood function.

For the KummerU PDF, the maximum log-likelihood function for segment S is derived from Eq. 8. The log-likelihood function can be rewritten as :

$$\begin{aligned}\mathrm{MLL}(S) =& -pN\ln(\pi) - N\ln\left\{|[\hat{M}_{ML}]|\right\} \\ &+ N\ln\left\{\frac{\Gamma(\hat{\mathcal{L}}+\hat{\mathcal{M}})\,\Gamma(p+\hat{\mathcal{M}})}{\Gamma(\hat{\mathcal{L}})\,\Gamma(\hat{\mathcal{M}})}\right\} + pN\ln\left\{\frac{\hat{\mathcal{L}}}{\hat{\mathcal{M}}\hat{m}}\right\} \\ &+ \sum_{\mathbf{z}_k\in S}\ln\left\{\mathrm{U}\left(p+\hat{\mathcal{M}}; 1+p-\hat{\mathcal{L}}; \frac{\hat{\mathcal{L}}}{\hat{\mathcal{M}}\hat{m}}\mathbf{k}_i^H[\hat{M}_{ML}]^{-1}\mathbf{k}_i\right)\right\}\end{aligned} \qquad (13)$$

where $\hat{\mathcal{L}}$, $\hat{\mathcal{M}}$ and $\hat{m}$ are respectively the ML estimators of the Fisher parameters $\mathcal{L}$, $\mathcal{M}$ and m. $[\hat{M}_{ML}]$ is the exact ML estimator of $[M_{ML}]$ for the segment S (Eq. 10).

3.3. Segmentation results

The hierarchical segmentation algorithm proposed by Beaulieu and Touzi [8] has been implemented for the Gaussian and KummerU criterion (Eq. 13) with high resolution airborne SLC images.

3.3.1. *On high resolution L-band data*

In this part, a forested area (500×400 pixels) over the Oberpfaffenhofen test-site (ESAR, L-band) has been segmented. The initial partition is composed of 2000 segments where each segment is a bloc of 10×10 pixels. Segmentation results with Gaussian and KummerU criterion are respectively shown on Fig. 2(a) and Fig. 2(b).

For a fixed number of segments in the final partition, the Gaussian criterion gives an over-segmented partition in textured scenes as forested areas. The use of a texture criterion allows to accurately segment heterogeneous scenes as forested areas.

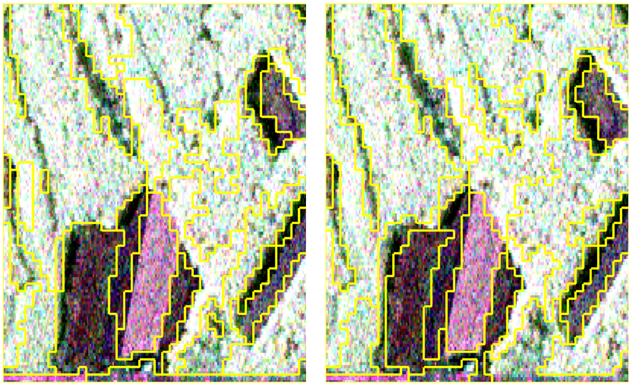
3.3.2. *On very high resolution X-band data*

Fig. 3(a) and Fig. 3(b) show respectively segmentation results over the Toulouse test-site with the Gaussian and KummerU criterion. This data-set has been acquired by the X-band RAMSES sensor with a resolution of 0.57m. The segmentation algorithm is initialized with a partition where each segment is a bloc of 10×10 pixels.

This example shows that for high-resolution data, heterogeneous clutter models should be use to accurately segment PolSAR data.

4. CONCLUSION

In this paper, authors have proposed to apply the SIRV estimation scheme to derive the covariance matrix and the tex-



(a) (b)

Fig. 2. Segmentation results for the L-band ESAR data over the Oberpfaffenhofen test-site (500×400 pixels). Partition containing 30 segments over a colored composition of the target vector $[k]_1$-$[k]_3$-$[k]_2$: (a) Gaussian criterion, (b) KummerU criterion

ture parameter. By rewriting the texture variable as the product of a mean backscattered power μ with a normalized texture component ξ, the Beta Prime PDF has been introduced to characterize the ξ variable. In this case, the texture parameter τ is Fisher distributed and the target scattering vector follows a KummerU PDF. Then, this distribution has been implemented in a ML hierarchical segmentation algorithm. Segmentation results on high resolution PolSAR data have shown that the SIRV estimation scheme combined with the KummerU PDF provide the best performances compared to the classical Gaussian hypothesis.

5. REFERENCES

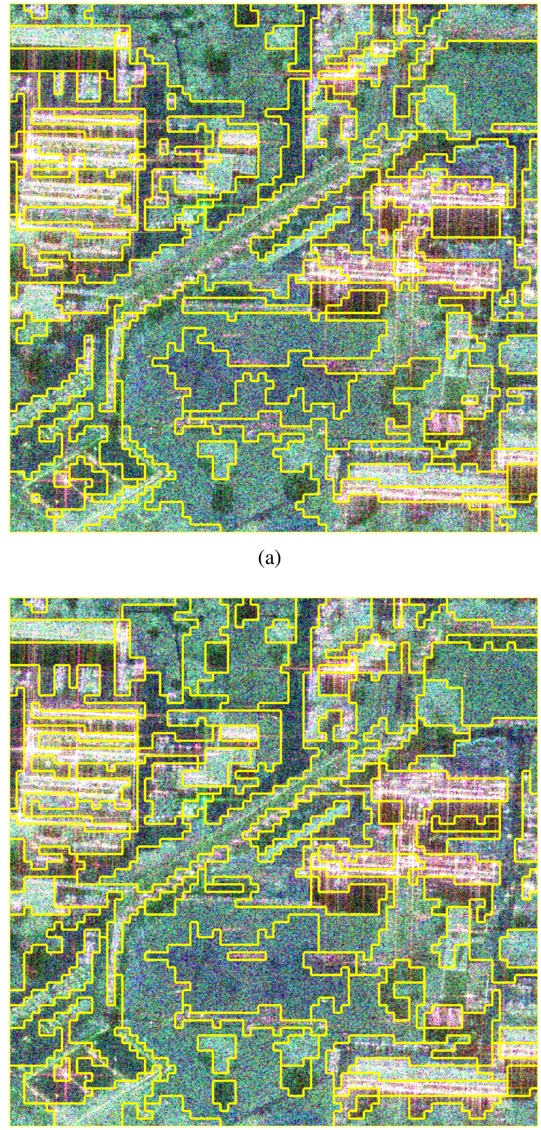
[1] K. Yao, "A representation theorem and its applications to spherically-invariant random processes," *IEEE Transactions on Information Theory*, vol. 19, no. 5, pp. 600–608, 1973.

[2] F. Pascal, Y. Chitour, J. P. Ovarlez, P. Forster, and P. Larzabal, "Covariance structure maximum-likelihood estimates in compound Gaussian noise: existence and algorithm analysis," *IEEE Transactions on Signal Processing*, vol. 56, no. 1, pp. 34–48, 2008.

[3] F. Pascal, P. Forster, J. P. Ovarlez, and P. Larzabal, "Performance analysis of covariance matrix estimates in impulsive noise," *IEEE Transactions on Signal Processing*, vol. 56, no. 6, pp. 2206–2216, 2008.

[4] J.-M. Nicolas, "Introduction aux Statistiques de deuxième espèce: applications des Logs-moments et des Logs-cumulants à l'analyse des lois d'images radar," *Traitement du Signal*, vol. 19, no. 3, pp. 139–167, 2002.

[5] F. Gini and M. V. Greco, "Covariance matrix estimation for CFAR detection in correlated heavy tailed clutter," *Signal Processing*, vol. 82, no. 12, pp. 1847–1859, 2002.



(a)

(b)

Fig. 3. Segmentation results for the X-band RAMSES data over the Toulouse test-site (700×700 pixels). Partition containing 100 segments over a colored composition of the target vector $[k]_1$-$[k]_3$-$[k]_2$: (a) Gaussian criterion, (b) KummerU criterion

[6] L. Bombrun and J.-M. Beaulieu, "Fisher Distribution for Texture Modeling of Polarimetric SAR Data," *IEEE Geoscience and Remote Sensing Letters*, vol. 5, no. 3, July 2008.

[7] Y. Chitour and F. Pascal, "Exact maximum likelihood estimates for SIRV covariance matrix: existence and algorithm analysis," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4563–4573, 2008.

[8] J.-M. Beaulieu and R. Touzi, "Segmentation of Textured Polarimetric SAR Scenes by Likelihood Approximation," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 42, no. 10, pp. 2063–2072, 0ctober 2004.