Statistical and geometrical tools for the classification of highly textured polarimetric SAR images

Pierre Formont
ONERA / SONDRA

PhD defense
Under the supervision of Frédéric Pascal, Jean-Philippe Ovarlez & Laurent Ferro-Famil
(PhD director)
Co-funded by the ONERA and the DGA

December 10, 2013
**Classification**

**Goal**

Sort pixels in a polarimetric SAR image in different groups thanks to their polarimetric properties, in an unsupervised way.

![Diagram]

- **Truth**
- **Measurements**
- **Observations**
- **Classification**
Outline

1. Introduction
2. Statistical context
3. Proposed framework
4. Statistical classification
5. Information geometry
6. Conclusions and perspectives
Outline

1 Introduction
   - Synthetic Aperture Radar
   - Statistics in SAR

2 Statistical context

3 Proposed framework

4 Statistical classification

5 Information geometry

6 Conclusions and perspectives
**Synthetic Aperture Radar**

**Principle of SAR**

1. **Plane trajectory**
2. **Pulses**
3. **Swath**

Measured signal: $k$ is a complex value.
Polarimetry

- **Polarization**: orientation of the electric field of the EM wave
- Several possible polarizations $\Rightarrow$ horizontal and vertical
- Monostatic configuration $\rightarrow S_{HV} = S_{VH}$.
- Measured signal: $\mathbf{k} = \begin{bmatrix} S_{HH} \\ \sqrt{2} S_{HV} \\ S_{VV} \end{bmatrix}$ is a complex vector of size $m = 3$. 

Reflected wave $E_R = \frac{e^{-jkr}}{r} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} E_I$

Incident wave $E_I$
Random modeling of the signal

- Interferences inside the resolution cells, non-stationarity, ... → model $k$ as a random variable.
- Common assumption: $k \sim \mathcal{CN}(0, T)$
  - Low resolution
  - Large number of scatterers in each resolution cell
  - Central Limit Theorem

- In high resolution images, number of scatterers in each resolution cell smaller → CLT not applicable.
- $k$ is no longer Gaussian-distributed

Need to model the non-Gaussianity

Introduction of a non-Gaussian model.
1. Introduction

2. Statistical context
   - Several models
   - Covariance matrix
   - The Fixed Point Estimator

3. Proposed framework

4. Statistical classification

5. Information geometry

6. Conclusions and perspectives
Several models

Non-Gaussian models for SAR

Previously proposed distributions:


The SIRV (Spherically Invariant Random Vectors) model

\[ k = \sqrt{\tau x} \]

- \(x\) (speckle): complex circular zero-mean Gaussian \(m\)-vector
- \(\tau\) (texture): positive random variable.

Several models

Non-Gaussian models for SAR

Previously proposed distributions:


The SIRV (Spherically Invariant Random Vectors) model

$$k = \sqrt{\tau} x$$

- $x$ (speckle): complex circular zero-mean Gaussian $m$-vector
- $\tau$ (texture): positive random variable.

Several models

Why choose this model?

- Takes into account the **heterogeneity** of the signal thanks to the texture $\tau$ (local variations of power).

- Contains **polarimetric information** in $x$ and $M = E[xx^H]$.

- Encompasses many different distributions: Gaussian, K distribution, Weibull, Cauchy, Student-t, Rice, etc, depending on the distribution of $\tau$.

- Provides a strong **unified framework**, notably for estimation purposes: e.g. covariance matrix estimator.
Traditionally, $k \sim \mathcal{CN}(0, T) \rightarrow$ need the covariance matrix $T = \mathbb{E}[kk^H]$.

**Problem**

$T$ unknown and only one observation of $k$

Estimation with neighbouring pixels.

Sample Covariance Matrix

$$\hat{T}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} k_i k_i^H \sim \mathcal{W}(T, N)$$
Traditionally, \( k \sim \mathcal{CN}(0, T) \) → need the covariance matrix \( T = \mathbb{E}[kk^H] \).

**Problem**

\( T \) unknown and only one observation of \( k \)

**Sample Covariance Matrix**

\[
\hat{T}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} k_i k_i^H \sim \mathcal{W}(T, N)
\]

- In the SIRV case, \( k = \sqrt{\tau}x \) with \( M = \mathbb{E}[xx^H] \).
- The Sample Covariance Matrix of the SIRV covariance matrix \( M \):

\[
\hat{T}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} k_i k_i^H = \frac{1}{N} \sum_{i=1}^{N} \tau_i x_i x_i^H \neq \frac{1}{N} \sum_{i=1}^{N} x_i x_i^H
\]
Under SIRV assumption, the \textbf{Approximate Maximum Likelihood Estimator} of the covariance matrix $\mathbf{M}$ is the solution of the following equation:

$$\hat{\mathbf{M}} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^H}{\mathbf{k}_i^H \hat{\mathbf{M}}^{-1} \mathbf{k}_i} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{x}_i \mathbf{x}_i^H}{\mathbf{x}_i^H \hat{\mathbf{M}}^{-1} \mathbf{x}_i}.$$ 

Called the \textbf{Fixed Point Estimator} $\hat{\mathbf{M}}_{FPE}$.

\textbf{Properties of the FPE}

- The solution \textbf{exists and is unique}, up to a scalar factor.
- It is \textbf{unbiased} and \textbf{consistent}.
- When $N$ is large: same asymptotic behavior as $\hat{\mathbf{M}}_{SCM}$ with a different secondary data number: $N$ for $\hat{\mathbf{M}}_{SCM}$, $\frac{m+1}{m}N$ for $\hat{\mathbf{M}}_{FPE}$. 

\textbf{Called the Fixed Point Estimator $\hat{\mathbf{M}}_{FPE}$}.

\textbf{Depends only on the speckle part of the signal}

\textbf{No corruption from the heterogeneous power.}
Outline

1. Introduction
2. Statistical context
3. Proposed framework
   - Wishart classifier
   - Illustration
4. Statistical classification
5. Information geometry
6. Conclusions and perspectives
Wishart classifier

Many existing techniques

Wishart classifier (K-means clustering)

- Initialization: \( P \) classes with class centers \( C_1, \ldots, C_P \)
- Reassignment:
  \[
  T \in \Omega_k \iff k = \arg \min_p \left( \ln |C_p| + \text{Tr} \left( C_p^{-1} T \right) \right) \quad \text{(Wishart distance: Lee, 1994)}
  \]
- Class center computation:
  \[
  C_k = \frac{1}{N} \sum_{T_i \in \Omega_k} T_i
  \]
Proposed framework

Many existing techniques

Wishart classifier (K-means clustering)

- Initialization: P classes with class centers $C_1, ..., C_P$
- Reassignment:
  \[ T \in \Omega_k \iff k = \arg\min_{p} (\ln |C_p| + \text{Tr} (C_p^{-1}T)) \] (Wishart distance: Lee, 1994)
- Class center computation:
  \[ C_k = \frac{1}{N} \sum_{T_i \in \Omega_k} T_i \]
Proposed framework

Many existing techniques

Wishart classifier (K-means clustering)

- Initialization: $P$ classes with class centers $C_1, ..., C_P$
- Reassignment:

  \[ T \in \Omega_k \Leftrightarrow k = \arg \min_p \left( \ln |C_p| + \text{Tr} \left( C_p^{-1} T \right) \right) \]  (Wishart distance: Lee, 1994)

- Class center computation:

  \[ C_k = \frac{1}{N} \sum_{T_i \in \Omega_k} T_i \]
Illustration

Dataset

Figure: Dataset, Brétigny

\[(1): \left( \frac{S_{HH} + S_{VV}}{\sqrt{2}}, \frac{S_{HH} - S_{VV}}{\sqrt{2}}, \sqrt{2}S_{HV} \right)\]
Limitation of the Gaussian assumption

(a) Using the SCM

(b) Using only the intensity

**Figure**: Wishart classification of the Brétigny area

⇒ same results with \( \text{Tr} (\mathbf{T}) \) and \( \mathbf{T} \) ?
**Influence of the SIRV assumption**

![Figure: Wishart classification of the Brétigny area](image)

- (a) Using the SCM
- (b) Using the FPE

*Figure: Wishart classification of the Brétigny area*

⇒ **better separation of heterogeneous areas**
Outline

1 Introduction

2 Statistical context

3 Proposed framework

4 Statistical classification
   - Motivations
   - Proposed approach
   - Box’s approximation
   - Applications

5 Information geometry

6 Conclusions and perspectives
Second step of the Wishart classifier

\[ T \in \Omega_k \iff k = \arg \min_p (\ln |C_p| + \text{Tr} \left(C_p^{-1} T\right)) \]

- No constraint on the minimum

- Difficulty finding an optimal number of classes.
Second step of the Wishart classifier

\[ T \in \Omega_k \iff k = \arg \min_p \left( \ln |C_p| + \text{Tr} \left( C_p^{-1} T \right) \right) \]

- No constraint on the minimum
- Difficulty finding an optimal number of classes.

Proposed approach: hypothesis test

Test if an hypothesis is valid and provides a threshold for the rejection of this hypothesis.
Goal

Compare the covariance matrices of two pixels $k^{(1)}$ and $k^{(2)}$.

Hypothesis test:

$$
\begin{align*}
H_0 : & \quad T_1 = T_2 = T, \\
H_1 : & \quad T_1 \neq T_2,
\end{align*}
$$

$T_1, T_2, T$ unknown $\Rightarrow$ estimated from $(k_1^{(1)}, \ldots k_{N_1}^{(1)})$ and $(k_1^{(2)}, \ldots k_{N_2}^{(2)})$

Generalized Likelihood Ratio Test

$$
\Lambda = \sup_{\theta} \frac{L(k; H_1, \theta)}{\sup_{\theta} L(k; H_0, \theta)} \overset{H_1}{\geq} \eta, \quad \text{where} \quad L(k; H, \theta) = \prod_{i} f(k_i|H, \theta).
$$
**Case $T_2$ known**

**GLRT**

$$\ln(\Lambda) = N_1 \left( \ln |T_2| - \ln |\hat{T}_1| + \text{Tr} \left( T_2^{-1} \hat{T}_1 \right) - m \right)$$

**For both SCM and FPE**

$$\ln(\Lambda) = d(\hat{T}_1, T_2) = \left( \ln |T_2| + \text{Tr} \left( T_2^{-1} \hat{T}_1 \right) \right) \Rightarrow \text{Wishart distance}$$

$\Rightarrow$ **Generalization of the Wishart distance**
Case where both matrices are unknown

GLRT

\[ \Lambda = \begin{vmatrix} \hat{T} \end{vmatrix}^{N_1+N_2}_{N_1} \begin{vmatrix} \hat{T}_1 \end{vmatrix}^{N_1}_{N_1} \begin{vmatrix} \hat{T}_2 \end{vmatrix}^{N_2}_{N_2} \exp \left( \text{Tr} \left( \hat{T}^{-1} \left[ N_1 \hat{T}_1 + N_2 \hat{T}_2 \right] \right) - (N_1 + N_2) m \right) \]

- SCM case:
  \[ \hat{T} = \frac{N_1 \hat{T}_1 + N_2 \hat{T}_2}{N_1 + N_2} \Rightarrow \Lambda = \begin{vmatrix} \hat{T} \end{vmatrix}^{N_1+N_2}_{N_1} \begin{vmatrix} \hat{T}_1 \end{vmatrix}^{N_1}_{N_1} \begin{vmatrix} \hat{T}_2 \end{vmatrix}^{N_2}_{N_2} \]

- FPE case:
  \[ \hat{T} = f(\hat{T}_1, \hat{T}_2) \]
**Box’s M-test (Gaussian case)**

**Bartlett’s distance (1937)**

\[
\Lambda_{Bar} = \frac{|\hat{T}_1|^{\nu_1 / 2} |\hat{T}_2|^{\nu_2 / 2}}{|\hat{T}|^{\nu / 2}}
\]

where \(\nu_i = N_i\) and \(\nu = N_1 + N_2\) are the degrees of freedom of the estimation of \(\hat{T}_i\) and \(\hat{T}\), respectively.

**Box’s \(\chi^2\) approximation (1949)**

\[
\Lambda_{Box} = -2(1 - c_1) \ln(\Lambda_{Bar}) \sim \chi^2 \left( \frac{1}{2} m(m + 1) \right)
\]

where \(c_1 = \left( \sum_{i=1}^{2} \frac{1}{\nu_i} - \frac{1}{\sum_{i=1}^{2} \nu_i} \right) \left( \frac{2m^2 + 3m - 1}{6(m + 1)} \right)\).
Box’s M-test (SIRV case)

Asymptotic property of the FPE: same asymptotic behavior as $\hat{M}_{SCM}$ with a different secondary data number: $N$ for $\hat{M}_{SCM}$, $\frac{m+1}{m}N$ for $\hat{M}_{FPE}$

Box’s $\chi^2$ approximation for the SIRV case

$$\Lambda_{Box} = -2(1 - c_1) \ln(\Lambda'_{Bar}) \sim \chi^2\left(\frac{1}{2} m(m+1)\right)$$

Difference from Gaussian case

$$\nu_i = \frac{m}{m+1}N_i \text{ and } \nu = \frac{m}{m+1}(N_1 + N_2).$$

Critical region

$$\Lambda_{Box} \overset{H_1}{\geq} \eta \Rightarrow C_r = \left\{ \Lambda_{Box}, \Lambda_{Box} \geq \eta = \chi^2_{PFA} \left(\frac{1}{2} m(m+1)\right) \right\}$$
Naive implementation

Initialization
1 class $\omega_1$ of center $C_1$

Increase number of classes

\forall \text{pixel}, \text{class}
Compute $\Lambda_{Box}(M_{\text{pixel}}, C_{\text{class}})$

If $\min_{C_{\text{class}}} \Lambda_{Box}(T_{\text{pixel}}, C_{\text{class}}) < \eta$
$\Rightarrow$ pixel in class

If $\min_{C_{\text{class}}} \Lambda_{Box}(T_{\text{pixel}}, C_{\text{class}}) > \eta$
$\Rightarrow$ pixel in rejection class

New class = rejection class
Recompute class centers

Stopping criterion reached ?

No

Yes

End
Naive implementation, classification results using the SCM

Figure: Classification results with SCM

(a) 1 iteration  
(b) 8 iterations
Naive implementation, classification results using the FPE

Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

![Hierarchical Clustering Diagram]

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{\text{Box}}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 

Figure: Hierarchical clustering
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

![Hierarchical Clustering Diagram]

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 

*Figure: Hierarchical clustering*
**Application to hierarchical clustering**

- Salembier and Alonso-Gonzalez (since 2010).

![Hierarchical Clustering Diagram](https://via.placeholder.com/150)

**Figure**: Hierarchical clustering

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

Figure: Hierarchical clustering

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

**Figure**: Hierarchical clustering

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 

Figure: Hierarchical clustering
Applications

Application to hierarchical clustering

- Salemier and Alonso-Gonzalez (since 2010).

![Hierarchical Clustering Diagram]

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
### Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

**Figure**: Hierarchical clustering

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

Figure: Hierarchical clustering

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Application to hierarchical clustering

- Salembier and Alonso-Gonzalez (since 2010).

![Hierarchical Clustering Diagram](image)

- Each pixel initially in its own class (leaf).
- At each iteration, merge closest pixels w.r.t. $\Lambda_{Box}$.
- Define a linkage function to merge clusters of pixels:
  - minimum distance
  - maximum distance
  - average distance
- Cut the tree at height given by the threshold $\eta$. 
Average distance

Figure: Hierarchical clustering results with average distance and $P_{FA} = 10^{-4}$

Outline

1 Introduction

2 Statistical context

3 Proposed framework

4 Statistical classification

5 Information geometry
   - Motivations
   - Theory
   - Application

6 Conclusions and perspectives
Motivations

Third step of the Wishart classifier

\[ C_k = \frac{1}{N} \sum_{T_i \in \Omega_k} T_i \]

Use the pixels of the class directly?

\[ C_k = \frac{1}{N_k} \sum_{n=1}^{N_k} k_n k_n^H \]

(a) Arithmetical mean  
(b) Estimation
Motivations

Third step of the Wishart classifier

$$C_k = \frac{1}{N} \sum_{T_i \in \Omega_k} T_i$$

Use the pixels of the class directly?

$$C_k = \frac{1}{N_k} \sum_{n=1}^{N_k} k_n k_n^H$$

(a) Arithmetical mean

(b) Estimation
Another way to look at the problem: consider the structure of the manipulated objects (covariance matrices) \( \Rightarrow \) Hermitian definite-positive matrices.

NOT Euclidean space: arithmetical mean not adapted to this space.

Euclidean mean (arithmetic)

\[
\arg \min_{M \in \mathcal{P}(m)} \sum_{i=1}^{N} d(M, M_i)^2, \text{ where } d(M, M_i) = \|M - M_i\|_F
\]

Riemannian mean (geometric)

\[
\arg \min_{M \in \mathcal{P}(m)} \sum_{i=1}^{N} d(M, M_i)^2, \text{ where } d(M, M_i) = ?
\]
Structure of covariance matrices

Euclidean mean (arithmetic)

\[ d(M, M_i) = \|M - M_i\|_F \]

Riemannian mean (geometric)

\[ d(M, M_i) = ? \]
Mean of Hermitian definite positive matrices

Riemannian distance between two matrices

\[ d(M_1, M_2)^2 = \left\| \log \left( \left( M_1^{-1/2} \right)^H M_2 M_1^{-1/2} \right) \right\|_F^2 \]

More convenient expression

\[ d(M_1, M_2) = \left\{ \sum_{k=1}^{n} \left( \log \lambda_k \right)^2 \right\}^{1/2} \]

No analytical expression for \( M! \)

\[ \sum_{i=1}^{N} \log \left( M_i^{-1} M \right) = 0. \]

Gradient descent algorithm

\[ M_{n+1} = \left( M_n^{1/2} \right)^H \exp \left( -\epsilon \sum_{i=1}^{N} \log \left( \left( M_n^{-1/2} \right)^H M_i^{-1} M_n^{-1/2} \right) \right) M_n^{1/2} \]
Moakher (2005) proposed a differential approach to compute the mean of symmetric positive-definite matrices.

Devlaminck (2010) demonstrated the added physical interpretation of a Riemannian mean for the covariance matrices in polarized light.

Wang (2010) used Riemannian geometry for PolSAR classification using the mean-shift algorithm.

Barbaresco (2010) proposed different approaches for the computation of the mean of Hermitian definite positive matrices and applications to radar signal processing, especially STAP processing.
Simulated data

Figure: Extraction of covariance matrices
Simulated data

(a) K-distributed data

(b) Power

Figure: Simulated data
Classification scheme

K-means clustering with 4 classes:

- Choice of Wishart distance or Riemannian distance
- Choice of Euclidean mean or Riemannian mean
- Choice of SCM or FPE
- Choice of supervised case (initial class centers are generating matrices $M_1, ..., M_4$) or unsupervised case (initial class centers are estimated through random initialization of the data).

Classification results, simulated data

All cases

SCM polluted by power.

(a) SCM

(b) Power
Classification results, simulated data

All cases (FPE, Euclidean mean)

Little difference between Wishart distance and Riemannian distance

(a) Riemannian distance

(b) Wishart distance
Classification results, simulated data

Supervised case (FPE, Wishart distance)

Little difference between Euclidean mean and Riemannian mean

(a) Euclidean mean
(b) Riemannian mean
Classification results, simulated data

Unsupervised case (FPE, Wishart distance)

Riemannian mean can perform better when matrices are not known

(a) Euclidean mean

(b) Riemannian mean
Classification scheme for real data

K-means clustering with 8 classes:

- Choice of Wishart distance or Riemannian distance
- Choice of Euclidean mean or Riemannian mean
- Fixed Point Estimator
- Choice of Cloude-Pottier initialization or random initialization.
Cloude-Pottier decomposition

Figure: Entropy - $\alpha$ plane
All cases

Little difference between Cloude-Pottier and random initialization.

(a) Cloude-Pottier

(b) Random

Figure: Euclidean mean, Wishart distance
Classification results, real data

All cases

Impact of Riemannian distance difficult to quantify.

Figure: Euclidean mean, Cloude-Pottier initialization

(a) Wishart distance

(b) Riemannian distance
Classification results, real data

Impact of Riemannian mean

Separates some features

Figure: Wishart distance, Cloude-Pottier initialization

(a) Euclidean mean

(b) Riemannian mean
Repartition in the $H - \alpha$ plane

(a) Classification results

(b) Repartition

Figure: SCM, Euclidean mean, Wishart distance
Repartition in the $H - \alpha$ plane

(a) Classification results
(b) Repartition

Figure: FPE, Euclidean mean, Wishart distance
Repartition in the $H - \alpha$ plane

(a) Classification results  
(b) Repartition

Figure: FPE, Riemannian mean, Wishart distance
Outline

1 Introduction

2 Statistical context

3 Proposed framework

4 Statistical classification

5 Information geometry

6 Conclusions and perspectives
   ▪ Conclusions
   ▪ Perspectives
Conclusions

Modeling

- Introduction of a non-Gaussian model for polarimetric SAR classification
- Limitations of the traditional Gaussian approach
- Unification of previous work
- Application on real data
- Increase interest of polarimetry
Conclusions

Statistical approach

- Original approach to the classification problem through hypothesis test
- Generalization of the traditional Wishart distance for the SIRV model
- Introduction of a rejection class
- Development of new algorithms and application on real data
Application of information geometry

- Introduction of tools for computation of mean of polarimetric covariance matrices
- Study impact on simulated data
- Application on real data
Perspectives

- Validation of all these techniques: physical interpretation
- Texture can provide polarimetric information jointly with covariance matrix
- Validity of the SIRV model: single texture for all polarisations?
- Local estimation of the covariance matrix
- Application to other data: hyperspectral, ...
Thanks for your attention